Problem Set 1, Due September 3

Feel free to do some experimentation via a computer algebra system to develop intuition for the problem.

1. If $z_1, \ldots, z_n$ are distinct complex numbers, show that the determinant

$$D = \begin{vmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_n \\ z_1^2 & z_2^2 & \cdots z_n^2 & \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{n-1} & z_2^{n-1} & \cdots & z_n^{n-1} \end{vmatrix}$$

   is nonzero. This may require a hint, so start it early. (10 points)
2. Use determinant from Problem 1 to show that the only polynomial that vanishes for all $z \in \mathbb{C}$ is the 0 polynomial. (3 points)
3. Let $n = 3$. Write down $\sum \alpha_i^k$ in terms of symmetric polynomials for $k = 1, 2, 3, 4$. Find a recursive formula for arbitrary k. In particular, if α, β, γ are the roots of the cubic $x^3 + px^2 + qx + r$ find a relation between $S_n := a^n + b^n + c^n$ and p, q, r. What do you think happens for n other than 3? (10 points)
4. If $f(x)$ has roots $\alpha_1, \cdots, \alpha_n$, what polynomial has roots $1/\alpha_1, \ldots, 1/\alpha_n$? (3 points)
5. How long, more or less, did it take you to complete this problem set? Any comments on it? (3 points)