Lab 1

Answer the following questions using Sage:

1: For various reason it would be nice to have a function $f(x)$ so that when you plugged some integer into f it gave only prime numbers.

- Investigate the polynomial $x^2 + x + 41$ keeping this in mind
- Investigate the polynomials $x^2 - 79x + 1601$ keeping this is mind
- Prove that a polynomial cannot only produce primes.
- Now weaken the question: Let $f(x) = x^2 + ax + b \in \mathbb{Z}[x]$ be a quadratic polynomial with integer coefficients. Formulate a conjecture about when the set

$$\{f(n) : n \in \mathbb{Z} \text{ and } f(n) \text{ is prime}\}.$$

  is infinite.

2: The Fibonacci numbers are defined recursively by $u_0 = 0$, $u_1 = 1$ and $u_{n+1} = u_n + u_{n-1}$.

- formulate a conjecture about $\gcd(u_a, u_b)$ in terms of some u_i.
- assume your conjecture and prove
  - u_n and u_{n+1} are relatively prime
  - if u_n is prime, then n is prime (also find a counterexample to the converse)
- prove your conjecture (this one might prove challenging)
- guess a formula for the inverse of $u_n \pmod{u_{n+1}}$
- prove your formula