Homework 8

This problem set is due April 10th.

Reading Finish Ch 3 by Wednesday

Problem 1 Do problem 3.4 in text book (you might want to base it on the proof of Lemma 2 on p 69)

Problem 2 Do 3.7(a). A useful group theory fact you may want to prove and use: if $\phi : G \to H$ is a homomorphism of commutative groups, show that if g has finite order, so does $\phi(g)$.

Problem 3 Let C be the curve $y^2 = x^3 + p^m x$ where p is prime and greater than 5 and $m \geq 1$. We're going to find all rational points of finite order:

(a) Find all possible primes p and integers m so that $p^m + 1$ is a perfect square.

(b) Find all torsion points of C (Problem 2 will probably be useful).

(c) What about for $p = 2, 3$? Find all rational points on $y^2 = x^3 + 64x$. Using Sage will only give you some points.