NAME (Print!): KEY

Quiz 6

Problem 1, 10 points: Let X_n be the random variable that equals the number of tails minus the number of heads when n coins are flipped.

(a) What is the expected value of X_n?

(b) What is the variance of X_n?

Let X_n denote the random variable.

Then decompose

$$X_n = X_1 + \cdots + X_n$$

where $X_i = \begin{cases} 1 & \text{if flip } i \text{ is T} \\ -1 & \text{" H} \end{cases}$

Then

(a) $E(X_n) = \sum E(X_i)$

$= \sum \frac{1}{2}(1) + \frac{1}{2}(-1) = 0.$

Note X_i, X_j are independent when $i \neq j$

(b) $V(X_n) = E(X_n^2) - E(X_n)^2$

$= E(X_n^2) - 0$ part (a)

$= E\left(\left(\sum X_i\right)^2\right)$

$= E\left(\sum X_i^2 + 2\sum X_i X_j\right)$ (multiplying it out)

$= \sum E(X_i^2) + 2\sum E(X_i)E(X_j)$ (X_i, X_j are indep)

$= n + 0$ (part a)

$= n.$