FINAL EXAM
MATH 211

DECEMBER 11, 2007

Name: ______________________

Professor (circle most appropriate): Dryden Gorkin Ryan No idea

You have 3 hours. Calculators, notes, textbooks and any other material are prohibited. There are 12 questions. Be sure to read all questions carefully. Justify all your answers, except where instructed otherwise.

Problem	Score	Out of
1		20
2		15
3		15
4		20
5		16
6		15
7		15
8		20
9		10
10		9
11		30
12		15

(20 pts.) Problem 1. Consider the function $f(x, y, z) = ze^{xy}$.

(a) Find the rate of change of f at the point $P(0, 1, 2)$ in the direction from P to $Q(2, 4, 3)$.

(b) Give a unit vector in the direction of the maximum rate of change of f.

(c) What is the maximum rate of change in the direction found in part (b)?

(15 pts.) Problem 2. Find parametric equations for the line in which the planes $x - 2y + 4z = 2$ and $x + y - 2z = 5$ intersect.

(15 pts.) Problem 3. Evaluate the integral: $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx$. Include a sketch of the region of integration.

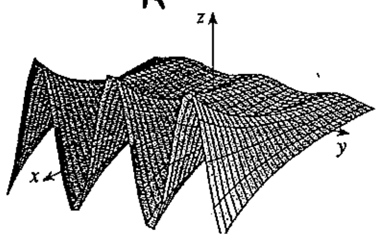
(20 pts.) Problem 4. Find the minimum and maximum of the function $f(x, y) = 4x^2 - xy + 4y^2$ over the closed disc $x^2 + y^2 \leq 2$.

(16 pts.) Problem 5.

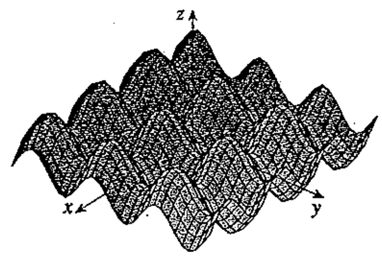
From the pictures below, choose a surface and a contour map that match each of the following equations:

(a) $z = e^x \cos y$
(b) $z = \sin(x - y)$

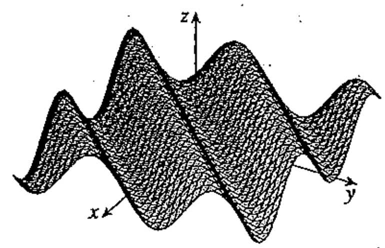
A



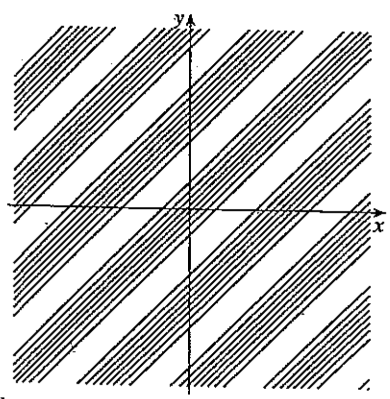
B



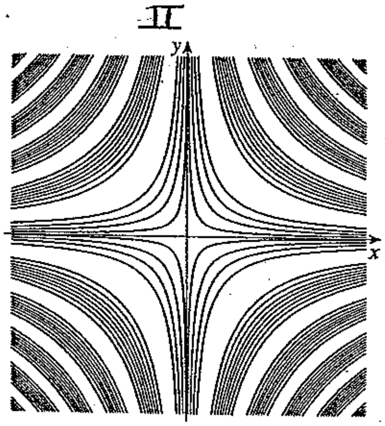
C



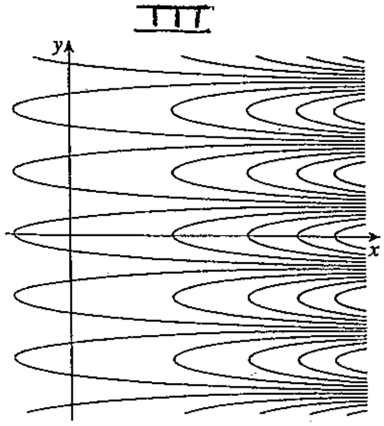
I



II



III



(15 pts.) Problem 6. For this problem, let $\mathbf{F}(x,y,z) = y^2 \cos z\mathbf{i} + 2xy\cos z\mathbf{j} + (\sin z - xy^2 \sin z)\mathbf{k}$.

(a) The vector field $\mathbf{F}$ above is conservative (you don't need to check this). Find the potential function f. (Do not guess what it is; compute. Provide explicit and careful calculations.)

(b) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ (where $\mathbf{F}$ is the same as above) along the curve $\mathbf{r}(t) = t^2\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ for $0 \le t \le \pi/3$. Show all work. (If you could not find a potential function in part (a) above, you may use $f(x,y,z) = x\tan z + yz\ln(x^2+1)$ as your potential function.)

(15 pts.) Problem 7. Evaluate the integral below:

$$\int_0^{1/2} \int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\,dy\,dx.$$

Include a sketch of the region of integration.

(20 pts.) Problem 8. Write an iterated triple integral for the integral of $f(x, y, z) = 6 + 4y$ over the region in the first octant bounded by the cone $z = \sqrt{x^2 + y^2}$, and the cylinder $x^2 + y^2 = 1$.

(a) cylindrical coordinates and

(b) spherical coordinates.

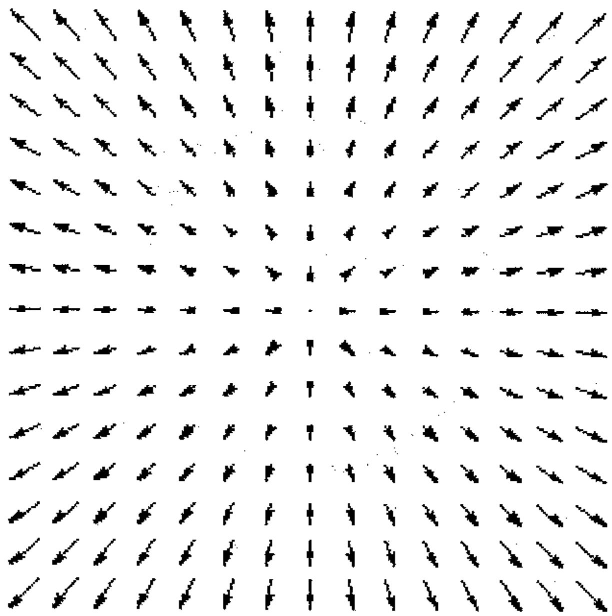
Do not integrate.

(10 pts.) Problem 9. Evaluate the limit below or explain why it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy + xy^2}{x^2 + y^2}.$$

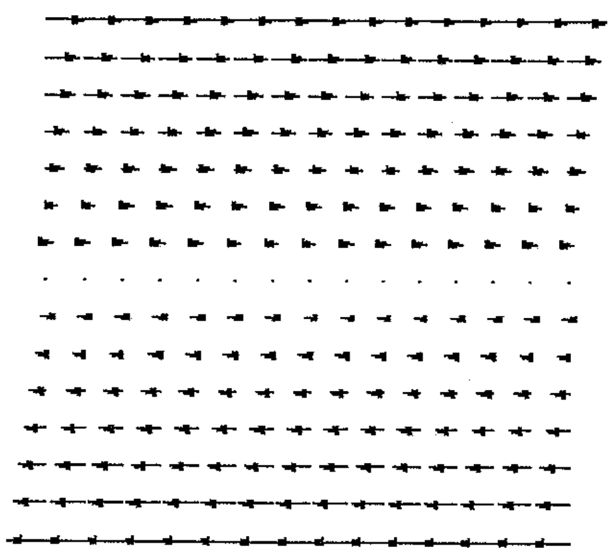
(9 pts.) Problem 10. (For this problem, there is no need to justify your answer.)

(a) Determine if the divergence at the central point is positive, negative or zero.



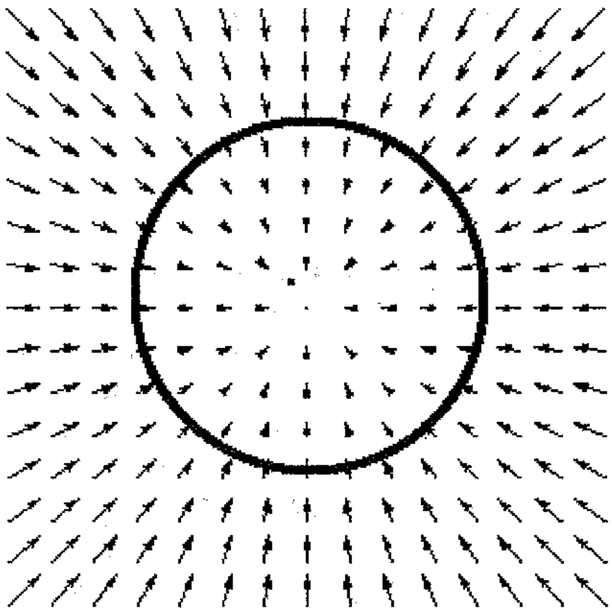
Ans: The divergence is ______________.

(a) Determine if the z-component of the curl (i.e., the **k**-component) is positive, negative or zero.



Ans: The z-component is ______________.

(b) Give the circle the counterclockwise orientation. Determine whether the line integral of the vector field is positive, negative or zero.



Ans: The integral is ______________.

(30 pts.) Problem 11. Let S be the surface defined by $x^2 + y^2 + 5z = 1$, $z \geq 0$, oriented by upward pointing normal. Verify Stokes's theorem in the case where

$$\mathbf{F}(x, y, z) = xz\mathbf{i} + yz\mathbf{j} + (x^2 + y^2)\mathbf{k}.$$

(15 pts.) Problem 12. Consider the cube that encloses a volume of 1 and is in the first octant with three faces on the xy-, yz- and xz-planes. Let S be the 5 faces of the cube that are not in the xy-plane. Also, let

$$\mathbf{F}(x, y, z) = ze^{z^2}\mathbf{i} + 3y\mathbf{j} + (2 - yz^7)\mathbf{k}.$$

Find the flux $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ of $\mathbf{F}$ across S. Do this in the simplest way possible.