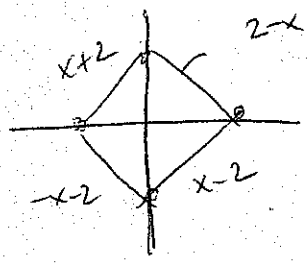
Quiz 6

Problem 1 Let D be the square in the xy-plane with vertices $(2,0)$, $(0,2)$, $(-2,0)$, and ~~$(2,0)$~~ $(0,-2)$ and consider the integral

$$\iint_D (x+y)\, dA.$$

(1) Evaluate this integral using iterated integration.



$$\int_{-2}^{0}\int_{-x-2}^{x+2} (x+y)\, dy\, dx + \int_{0}^{2}\int_{x-2}^{2-x} (x+y)\, dy\, dx$$

$$= \int_{-2}^{0} \left(xy+\frac{y^2}{2}\right)\Big|_{-x-2}^{x+2} dx + \int_{0}^{2} \left(xy+\frac{y^2}{2}\right)\Big|_{x-2}^{2-x} dx$$

$$= \int_{-2}^{0} x(x+2) + \frac{(x+2)^2}{2} - \left(x(-x-2) + \frac{(-x-2)^2}{2}\right) dx$$

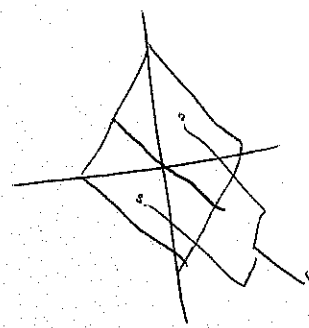
$$+ \int_{0}^{2} x(2-x) + \frac{(2-x)^2}{2} - \left(x(x-2) + \frac{(x-2)^2}{2}\right) dx$$

$$= \int_{-2}^{0} 2x^2 + 4x\, dx + \int_{0}^{2} 4x - 2x^2\, dx$$

$$= \int_{-2}^{2} 4x\, dx$$

$$= 0.$$

(2) Evaluate this integral using only the properties of double integrals.



These contributions cancel out to 0 and so, by symmetry, the integral is 0.