Quiz 5

Problem 1 The maximum rate of change of $f(x, y) = x^3y^4$ at $(1, 1)$ is

Max rate of change given by $\|\nabla f_p\| = \| \langle 3x^2y^4, 4x^3y^3 \rangle_{(x,y)=(1,1)} \|$

$= \| \langle 3, 4 \rangle \|$

$= \boxed{5.}$

Problem 2 The directional derivative of the function $9\sqrt{x^2 + y^2 + z^2}$ at $(1, 2, -2)$ in the direction of $\mathbf{v} = \langle -6, 6, -3 \rangle$ is

$$\nabla f = \left\langle \frac{9x}{\sqrt{x^2+y^2+z^2}}, \frac{9y}{\sqrt{x^2+y^2+z^2}}, \frac{9z}{\sqrt{\ }} \right\rangle_{(1,2,-2)} = \left\langle \frac{9}{3}, \frac{18}{3}, \frac{-18}{3} \right\rangle = \langle 3, 6, -6 \rangle$$

$$\vec{u} = \frac{\langle -6, 6, 3 \rangle}{\sqrt{6^2+6^2+3^2}} = \left\langle -\frac{6}{9}, \frac{6}{9}, \frac{-3}{9} \right\rangle = \left\langle -\frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \right\rangle$$

$$\nabla f \cdot \vec{u} = \cancel{8}\ -2 + 4 + 2 = \boxed{4}$$

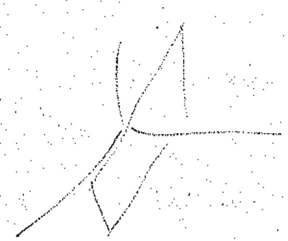
Problem 3 Identify the surface whose equation in spherical coordinates is given by $\rho = \cos\phi$.

$\rho = \cos\varphi \Rightarrow \rho^2 = \rho\cos\varphi \Rightarrow x^2 + y^2 + z^2 = z$

$\Rightarrow x^2 + y^2 + \left(z - \frac{1}{2}\right)^2 = \frac{1}{4}$

sphere of radius $\frac{1}{2}$ centered at $(0, 0, \frac{1}{2})$

Problem 4 Let C be the curve resulting from the intersection of the plane $y = 2$ with the surface of the function $f(x, y) = 2x^3 + 3x^2y - 2xy^2$. Find the slope of the tangent line to C at the point $(1, 2, 0)$.



This is just $\frac{\partial f}{\partial x}$ at $(1, 2, 0)$.

$\frac{\partial f}{\partial x} = 6x^2 + 6xy - 2y^2 \quad \left.\frac{\partial f}{\partial x}\right|_{(1,2,0)} = 6 + 12 - 8$

$= 10$

Problem 5 Use the linear approximation of $f(x, y) = ye^{3x-2y}$ at $(2, 3)$ to approximate the value of $f(2.01, 2.99)$.

$f(2 + 0.01, 3 - 0.01) \approx f(2,3) + f_x(2,3)(0.01) + f_y(2,3)(-0.01)$

$$\boxed{\begin{array}{l} f_x = 3ye^{3x-2y}, \quad f_x(2,3) = 9 \\ f_y = e^{3x-2y} - 2ye^{3x-2y}, \quad f_y(2,3) = 1 - 6 = -5. \end{array}}$$

$= 3 + 9(.01) - 5(-0.01) = 3.14.$