Math 211

March 8, 2012
Sample Second Midterm

NAME: _____

Problem	Points	Score
1	5	
2	9	
3	10	
4	10	
5	6	
6	10	
Total	50	

Problem 1 (5 points): Answer the following True and False questions. You will get 1 point for a correct answer, lose 1 point for a wrong answer and will neither lose nor get a point for leaving it blank. Write out the entire word: TRUE or FALSE

(1) $\lim_{(x,y)\to(0,0)} \frac{x^2-3xy+y^2}{(x-y)^2} = 1$

(2) The level curves for the surface $z = y^2 - x^2$ are hyperbolas

(3) If the tangent plane to the graph of $f(x, y)$ at the point $(x_0, y_0, f(x_0, y_0))$ is horizontal, then $(\nabla f)(x_0, y_0) = \vec{0}$.

(4) Consider the line $\vec{\ell}(t) = (1 + t, 2 - t, 3 + 2t)$. All the following planes
   (a) $x + y = 6$
   (b) $x + y = 5$
   (c) $2y + z = 6$
   (d) $2y + z = 5$
   (e) $2x + 4y + z = 0$

   are parallel to $\vec{\ell}(t)$.

(5) Let $f(x, y) = e^{x^2 \sin(y)}$. Then $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ at the point $(1, \pi/4)$.

Problem 2 (9 points): Find these derivatives:

(1) Let $f(x, y, z) = x^2 + y^2 + z^2$ and $\vec{c}(s, t) = (s \cos t, e^{st}, s^2 - t^2)$. Find $\frac{\partial f}{\partial s}$.

(2) Consider $f(x, y, z) = 5x^2 + 7y^4 + x^2z^2$. Find the rate of change of f in the direction $\langle 1, 1, 1 \rangle$.

(3) Consider $x^2/4 + y^2/36 + z^2/9 = 1$. Find $\frac{\partial z}{\partial x}$.

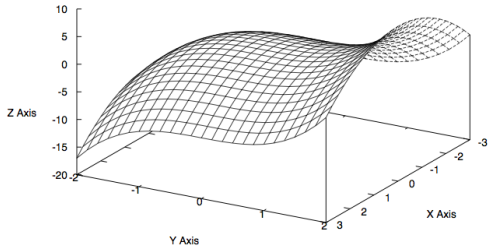
Problem 3 (10 points): Questions about curves on surfaces:

(1) The plane $y = 1$ intersects the graph of $z = 3x^2y - xy^3 + 5y^2$ in a curve. The tangent line to this curve at the point $(1, 1, 7)$ passes through a point $(0, 1, c)$. What is c?

(2) A surface S is given by $z = 4 - x^2 - y^2$.
   (a) Find the equation of the tangent plane of S at the point $(1, 1, 2)$.
   (b) Let C be the curve where S intersects the surface defined by $x^2y = 1, x > 0$. Think of C as a trail in a terrain described by S. How steep is the trail at the point lying directly above $(x, y) = (1, 1)$? Give the answer as an angle related to the horizontal plane.

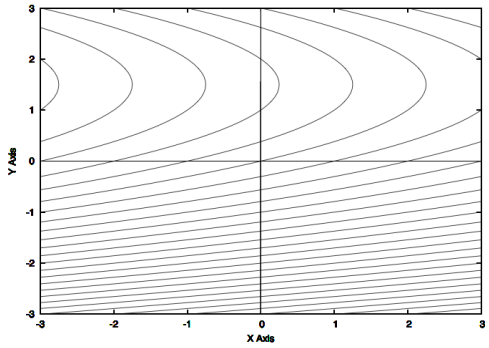
Problem 4 (10 points): Some graphing problems:

(1) Match the graph to the correction function:



(a) $f(x,y) = y^2$
(b) $f(x,y) = x^2$
(c) $f(x,y) = x - y^2$
(d) $f(x,y) = y - x^2$
(e) $f(x,y) = y^3 - x^2$
(f) $f(x,y) = x^2 - y3$
(g) $f(x,y) = y^4 - x^2$
(h) $f(x,y) = x^4 - y^2$
(i) $f(x,y) = y^4 - x$
(j) $f(x,y) = x^4 - y$

(2) Match the level curves to the correct function



(a) $f(x,y) = x^2$
(b) $f(x,y) = y^2$
(c) $f(x,y) = x^2 - y^2$
(d) $f(x,y) = x^2 + y^2$
(e) $f(x,y) = x + y^2$
(f) $f(x,y) = y + x^2$
(g) $f(x,y) = y + x^2 + 3x$
(h) $f(x,y) = y + x^2 - 3x$
(i) $f(x,y) = x + y^2 + 3y$
(j) $f(x,y) = x + y^2 - 3y$

Problem 5 (6 points): Suppose that the magnitude of two vectors are measured as 5 and 6 respectively and the maximum error in the measurement of each is 0.4. Suppose that the angle between the two of them is measured at $\pi/3$ radians with the maximum error in that measurement being 0.02 radians. Use linear approximation to find the maximum error in computing the dot product of the two vectors using the given data.

Problem 6 (10 points): Let $g(x, y) = \sqrt[3]{xy}$.

(1) Is g continuous at $(0, 0)$?

(2) Calculate g_x and g_y when $xy \neq 0$.

(3) Show that $g_x(0, 0)$ ad $g_y(0, 0)$ exist by thinking about the (geometric) definition of the partial derivatives.

(4) Are g_x and g_y continuous at $(0, 0)$?

(5) Does the graph of $z = g(x, y)$ have a tangent plane at $(0, 0)$?

(6) Is g differentiable at $(0, 0)$?