Math 211

February 9, 2012

Sample First Midterm

NAME: ______________________________ 1pm or 2pm__________

Problem	Points	Score
1	5	
2	10	
3	8	
4	10	
5	7	
6	10	
Total	50	

Problem 1 (5 points): Answer the following True and False questions. You will get 1 point for a correct answer, lose 1 point for a wrong answer and will neither lose nor get a point for leaving it blank. Write out the entire word: TRUE or FALSE

(1) Exactly two of the following expressions don't make sense:

$$||\mathbf{v}||\mathbf{w} - ||\mathbf{w}||\mathbf{v} \qquad (\mathbf{v}-\mathbf{w})\cdot \mathbf{u}/2 \qquad \frac{\mathbf{v}\cdot\mathbf{w}}{\mathbf{u}} \qquad \frac{\mathbf{v}}{\mathbf{v}\cdot\mathbf{w}}$$

OK OK Not OK

False

(2) The lines

$$\mathbf{r}_1(t) = \langle -\tfrac{t}{2}, \tfrac{2}{3}t+1, t+\tfrac{3}{2}\rangle$$
$$\mathbf{r}_2(r) = \langle t-\tfrac{3}{2}, 2t+3, 3t+\tfrac{9}{2}\rangle$$

intersect.

False (maybe an alg mistake)

(3) The arclength parameterization for the circle of radius a is given by

$$\mathbf{r}(t) = \langle a\cos(\tfrac{s}{a}), a\sin(\tfrac{s}{a})\rangle.$$

$r'(t) = \langle -\sin(\frac{s}{a}), \cos(\frac{s}{a})\rangle$ Speed 1.

TRUE

(4) Suppose $\mathbf{v} \neq \mathbf{w}$. Then

$$\mathbf{v}_\perp \neq w_\perp.$$

if $\mathbf{v} \perp \mathbf{w}$, $\mathbf{v}_\perp = \vec{0}$, $w_\perp = \vec{0}$

F

False

(5) Let $\mathbf{r}(t)$ describe the path taken by a particle. Then $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}''(t)$.

False

$\langle -\frac{t}{2}, \frac{2}{3}t+1, t+\frac{3}{2}\rangle = \langle s-\frac{3}{2}, 2s+3, 3s+\frac{9}{2}\rangle$

$-\frac{t}{2} = s - \frac{3}{2} \Rightarrow t = -2s+3$

$\frac{2}{3}t+1 = 2s+3$

$\frac{2}{3}(-2s+3) = 2s+3$

$-\frac{4}{3}s + 2 = 2s+3$

$-1 = \frac{10}{3}s$

$\frac{-3}{10} = s$

$-\frac{t}{2} = \frac{-3}{10} - \frac{3}{2}$

$t = \frac{18}{5}$

$\frac{18}{5} + \frac{3}{2} = \frac{-9}{10} + \frac{9}{2}$

$\frac{36+15}{10} = \frac{-9}{}$

(5) $\langle \cos t, \sin t\rangle$

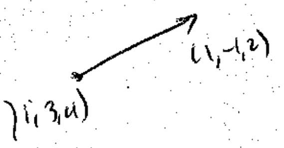
$\langle -\sin t, \cos t\rangle$

$\langle -\cos t, -\sin t\rangle$

$-\cos^2 t - \sin^2 t = -1$

Problem 2 (10 points): Find a parameterization for the following. curves:

(1) The line segment from the point $(1,3,4)$ to the point $(1,-1,2)$ going in that direction.



$r(t) = \langle 1,3,4\rangle + t\langle 0,-4,-2\rangle$

$0 \leq t \leq 1$

(2) The circle parallel to the yz-plane, of radius 5, with center $(1,1,1)$.

@ origin $\langle 0, 5\cos t, 5\sin t\rangle$

shifted

$\langle 1, 1+5\cos t, 1+5\sin t\rangle$

Problem 3 (8 points): Find the volume of the parallelepiped spanned by the vectors

$$\mathbf{u} = \langle 1, 2, -3 \rangle,\ \mathbf{v} = \langle 1, -2, 1 \rangle,\ \mathbf{w} = \langle -1, -2, -1 \rangle.$$

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Also, do the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ form a right-handed or left-handed coordinate system.

$$u \cdot (v \times w)$$

$$v \times w = \det \begin{pmatrix} i & j & k \\ 1 & -2 & 1 \\ -1 & -2 & -1 \end{pmatrix}$$

$$= i(4) - j(0) + k(-2 \ -2)$$

$$= \langle 4, 0, -4 \rangle$$

$$u \cdot (v \times w) = \langle 1, 2, -3 \rangle \cdot \langle 4, 0, -4 \rangle$$

$$= +16.$$

So the volume is 16 and it's a right hand system because $u \cdot (v \times w) > 0$.

Problem 4 (10 points): Suppose

$$\mathbf{a}(t) = \langle t+1, -t^2, 2t+3\rangle,\ \mathbf{r}(0) = \langle 1,0,1\rangle, \mathbf{v}(0) = \langle 0,0,0\rangle.$$

Find the position function $\mathbf{r}(t)$ that satisfies these conditions.

$$v(t) = \left\langle \frac{1}{2}t^2 + t,\ \frac{-t^3}{3},\ t^2 + 3t \right\rangle + \underbrace{\vec{v}_0}_{=\langle 0,0,0\rangle}$$

$$r(t) = \cancel{\langle \tfrac{1}{2}t^2}$$

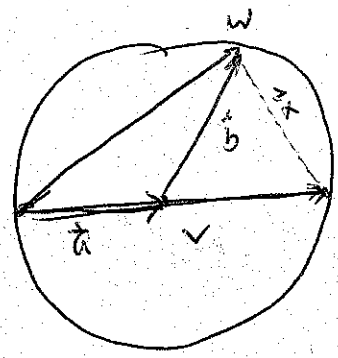
$$= \left\langle \frac{1}{3}t^3 + \frac{1}{2}t^2,\ \frac{-t^4}{12},\ \frac{t^3}{3} + \frac{3}{2}t^2 \right\rangle + \vec{r}_0$$

$$= \left\langle \frac{1}{3}t^3 + \frac{1}{2}t^2 + 1,\ \frac{-t^4}{12},\ \frac{t^3}{3} + \frac{3}{2}t^2 + 1 \right\rangle.$$

Problem 5 (7 points): Prove, using vectors, that if A and B are endpoints of a diameter of a circle, then $\triangle ABC$ is a right triangle.

C is any point on the circle

~~w·(v-a)~~



$$\vec{w} = \vec{a} + \vec{b}$$
$$\vec{x} = \vec{a} - \vec{b}.$$

I want $\vec{w} \cdot \vec{x} = \vec{a} \cdot \vec{a} - \vec{b} \cdot \vec{b}$

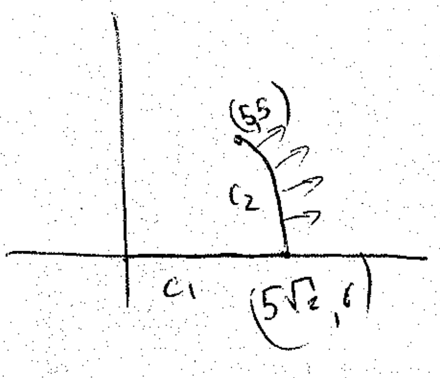
$= \|a\|^2 - \|b\|^2$

$= 0$ since $\vec{a}$, and $\vec{b}$ are both radii of the circle.

Problem 6 (10 points): Let C be the path from $(0,0)$ to $(5,5)$ consisting of the straight line from $(0,0)$ to $(5,\sqrt{2})$ and the circular arc from $(5,\sqrt{2},0)$ to $(5,5)$. Compute the total work done by the vector field

$$\mathbf{F}(x,y) = \langle x, y\rangle$$

on a particle moving along C. For full credit, you have to exploit geometry whenever possible.



$$\int_C = \int_{C_1} + \int_{C_2}$$

but $\int_{C_2} \vec{F}\cdot d\vec{s} = 0$

Since $F \perp d\vec{s}$ along that circle.

So

$c(t) = \langle t, 0\rangle$ $(0 \le t \le 5\sqrt{2})$

$$\int_C \vec{F}\cdot d\vec{s} = \int_{C_1} \vec{F}\cdot d\vec{s}$$

$$= \int_0^{5\sqrt{2}} \langle t, 0\rangle \cdot \langle 1, 0\rangle \, dt$$

$$= \frac{t^2}{2}\Big|_0^{5\sqrt{2}} = \boxed{25.}$$