Math 211

February 9, 2012
Sample First Midterm

NAME: ______________________________ 1pm or 2pm__________

Problem	Points	Score
1	5	
2	10	
3	8	
4	10	
5	7	
6	10	
Total	50	

Problem 1 (5 points): Answer the following True and False questions. You will get 1 point for a correct answer, lose 1 point for a wrong answer and will neither lose nor get a point for leaving it blank. Write out the entire word: TRUE or FALSE

(1) Exactly two of the following expressions don't make sense:

$$||\mathbf{v}||\mathbf{w} - ||\mathbf{w}||\mathbf{v} \qquad (\mathbf{v} - \mathbf{w}) \cdot \mathbf{u}/2 \qquad \frac{\mathbf{v} \cdot \mathbf{w}}{\mathbf{u}} \qquad \frac{\mathbf{v}}{\mathbf{v} \cdot \mathbf{w}}$$

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(2) The lines

$$\mathbf{r}_1(t) = \langle -\tfrac{t}{2}, \tfrac{2}{3}t + 1, t + \tfrac{3}{2} \rangle$$
$$\mathbf{r}_2(r) = \langle t - \tfrac{3}{2}, 2t + 3, 3t + \tfrac{9}{2} \rangle$$

intersect.

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(3) The arclength parameterization for the circle of radius a is given by

$$\mathbf{r}(t) = \langle a\cos(\tfrac{s}{a}), a\sin(\tfrac{s}{a}) \rangle.$$

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(4) Suppose $\mathbf{v} \neq \mathbf{w}$. Then

$$\mathbf{v}_\perp \neq w_\perp.$$

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(5) Let $\mathbf{r}(t)$ describe the path taken by a particle. Then $\mathbf{r}(t)$ is orthogonal to $\mathbf{r}''(t)$.

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Problem 2 (10 points): Find a parameterization for the following. curves:

(1) The line segment from the point $(1, 3, 4)$ to the point $(1, -1, 2)$ going in that direction.

(2) The circle parallel to the yz-plane, of radius 5, with center $(1, 1, 1)$.

Problem 3 (8 points): Find the volume of the parallelepiped spanned by the vectors

$$\mathbf{u} = \langle 1, 2, -3 \rangle,\ \mathbf{v} = \langle 1, -2, 1 \rangle,\ \mathbf{w} = \langle -1, -2, -1 \rangle.$$

Also, do the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ form a right-handed or left-handed coordinate system.

Problem 4 (10 points): Suppose

$$\mathbf{a}(t) = \langle t+1, -t^2, 2t+3 \rangle,\ \mathbf{r}(0) = \langle 1, 0, 1 \rangle, \mathbf{v}(0) = \langle 0, 0, 0 \rangle.$$

Find the position function $\mathbf{r}(t)$ that satisfies these conditions.

Problem 5 (7 points): Prove, using vectors, that if A and B are endpoints of a diameter of a circle and C is any endpoint, then $\triangle ABC$ is a right triangle. Hint: let $\mathbf{v}$ be $\vec{BA}$ and $\mathbf{w}$ be $\vec{AC}$. Write $\mathbf{w}$ as a sum of two vectors, one that has the center as its tail and the other that has the center as its head.

Problem 6 (10 points): Let C be the path from $(0,0)$ to $(5,5)$ consisting of the straight line from $(0,0)$ to $(5\sqrt{2}),0$ and the circular arc from $(5,\sqrt{2},0)$ to $(5,5)$. Compute the total work done by the vector field

$$\mathbf{F}(x,y) = \langle x, y \rangle$$

on a particle moving along C. For full credit, you have to exploit geometry whenever possible.