Math 211

April 9, 2012

Third Midterm (part 2)

NAME: KEY

| Problem | Points | Score |
|---|---|---|
| 4 | 5 | |
| 5 | 10 | |
| 6 | 10 | |
| Total | 25 | |

**Problem 1 (5 points):** Answer the following True and False questions. You will get 1 point for a correct answer, lose 1 point for a wrong answer and will neither lose nor get a point for leaving it blank. Write out the entire word: TRUE or FALSE. When appropriate, some work (or argument) must be shown, you can't just do it on your calculator.

(1) The mass of a cylindrical surface of radius $r = 3$ centered on the $z$-axis and bounded by the planes $z = 1$ and $z = 3$ if the density function is equal to the distance to the $xy$-plane is $24\pi$.

$36\pi$ ?

FALSE

(2) Let $D = [0,1] \times [0,2]$. Then $\iint_D e^{x^2} - e^{y^2}\, dA > 0$.

FALSE

(3) Let $D = [-2,0] \times [-2,0] \cup [0,2] \times [0,2]$. Then $\iint_D \sin(x) cos(y/2)\, dA > 0$.

$\geq 0$

FALSE

(4) Let $\vec{F} = \langle x, y, z \rangle$ and $S$ be the sphere of radius $R$ with the outward orientation. Then $\iint_S \vec{F} \cdot d\vec{S} > 0$.

TRUE

(5) If a surface $S$ lies completely below the plane $z = 0$ then $\iint_S dS < 0$.

$>$

TRUE

$$\iiint_W z\, dV$$

$$\int_0^{2\pi}\int_0^3\int_1^3 z \cdot r\, dz\, dr\, d\theta$$

$$\frac{z^2}{2}\Big|_1^3$$

$$4$$

$$4\int_0^{2\pi}\int_0^3 r\, dr\, d\theta$$

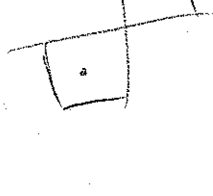
$$4\int_0^{2\pi} \frac{9}{2}\, d\theta$$

$$\iint_D e^{x^2} - e^{y^2}\, dA = \int_0^2\int_0^1 e^{x^2} - e^{y^2}\, dx\, dy$$

$$= \int_0^1\int_0^1 e^{x^2} - e^{y^2}\, dx\, dy + \int_1^2\int_0^1 e^{x^2} - e^{y^2}\, dx\, dy$$

$= 0 +$ something negative

$$\int_{-2}^0\int_{-2}^0 \sin x \cos\frac{y}{2}\, dx\, dy = \int_0^2\int_0^2 \sin x \cos\frac{y}{2}$$

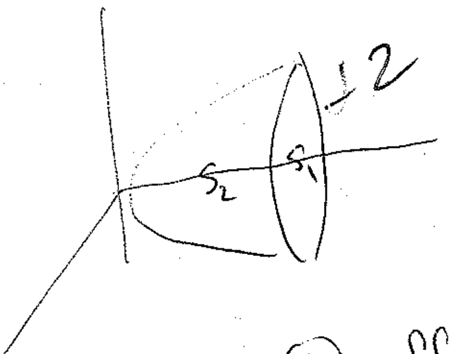


$$\iint \frac{\langle x,y,z\rangle \cdot \langle x,y,z\rangle}{R}\, dS$$

$$= \iint dS$$

$$= 4\pi R^2.$$

**Problem 2 (10 points):** Evaluate $\iint \vec{F} \cdot d\vec{S}$ where $\vec{F} = \langle 0, y, -z \rangle$ and S is the surface given by the paraboloid $y = x^2 + z^2$, and the disk $x^2 + z^2 \leq 1$ at $y = 1$. Assume that S has ~~positive~~ outward (positive) orientation. Feel free to compute the double integral by calculator.




$$\iint_S \vec{F} \cdot d\vec{S} = \iint_{S_1} + \iint_{S_2}$$

① $$\iint_{S_1} \vec{F} \cdot d\vec{S} = \iint_{S_1} (\vec{F} \cdot \hat{j})\, dS = \iint_{S_1} y \, dS$$
$$= \iint_{S_1} dS$$
$$= \text{Area}(S_1)$$
$$= \pi$$

+3

② $$\iint_{S_2} \vec{F} \cdot d\vec{S} = \iint_{S_2} (\vec{F} \cdot \vec{n})\, dS = \iint_D \langle 0, y, -z \rangle \cdot \langle 2x, -1, 2z \rangle \, dA$$

$$= \iint_{S_2}$$

$$\cancel{G(u,v) = \langle u, u^2+v^2, v \rangle}$$

$$= \iint_D -y - 2z^2 \, dA$$
$$= \iint_D -(x^2+z^2) - 2z^2 \, dA$$
$$= \iint_D -x^2 - 3z^2 \, dA$$

$$= -\iint_D x^2 + 3z^2 \, dA = -\int_0^{2\pi}\int_0^1 (r^2\cos^2\theta + 3r^2\sin^2\theta)\, r\,dr\,d\theta$$

$$= -\pi$$

Thus $$\iint_S \vec{F} \cdot d\vec{S} = 0.$$

**Problem 3 (10 points):** Consider the surface $S$ that is the part of the cone $z = \sqrt{x^2+y^2}$ below the plane $z = 3$.

(1) Give a parametric representation of $S$. Make sure to explicitly describe or sketch the parametrization domain $D$.

D disk of radius 3 centered at (0,0)


$G(u,v) = \langle u, v, \sqrt{u^2+v^2} \rangle$

5

$\frac{1}{\sqrt{2}}$

(2) Find an equation of the tangent plane to $S$ at the point $P(1, 1, \sqrt{2})$.

$G_u = \langle 1, 0, \frac{1}{2} \cdot 2u\,(u^2+v^2)^{-1/2} \rangle$ @P $\Rightarrow$ $G_u(1,1) = \langle 1, 0, \frac{1}{\sqrt{2}} \rangle$

$G_v = \langle 0, 1, \frac{1}{2}\, 2v\,(u^2+v^2)^{-1/2} \rangle$ @P $\Rightarrow$ $G_v(1,1) = \langle 0, 1, \frac{1}{\sqrt{2}} \rangle$

5

normal vector:

$$\det \begin{pmatrix} i & j & k \\ 1 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & \frac{1}{\sqrt{2}} \end{pmatrix} = i\,\frac{-1}{\sqrt{2}} - j\left(\frac{1}{\sqrt{2}}\right) + 1$$

$= \langle \frac{-1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 1 \rangle$.

So plane is

$$\boxed{\frac{-1}{\sqrt{2}}(x-1) - \frac{1}{\sqrt{2}}(y-1) + (z-\sqrt{2}) = 0}$$

$-x - y + \sqrt{2}z = 0$

$-(x-1) - (y-1) + \sqrt{2}z - 2 = 0$

$-x + 1 - y + 1 + \sqrt{2}z - 2 = 0$