Math 211

March 8, 2012

~~Sample~~ Second Midterm

NAME: ______________________________

Problem	Points	Score
1	5	
2	10	
3	10	
4	10	
5	5	
6	10	
Total	50	

Problem 1 (5 points): Answer the following True and False questions. You will get 1 point for a correct answer, lose 1 point for a wrong answer and will neither lose nor get a point for leaving it blank. Write out the entire word: TRUE or FALSE

(1) The level curves of a function of two variables can intersect each other.

False

(2) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are continuous, then f is differentiable.

check all hypotheses

True

(3) If $f(x,y)$ is continuous as a function of x for each fixed y and continuous as a function of y for each fixed x, then f is continuous.

False

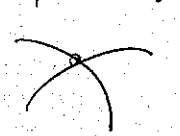
(4) If $f(x,y,z)$ is a differentiable function, then the direction of most rapid decrease is ∇f.

False

(5) Consider the function $f(x,y) = y^2 - x^2 - yx^3$. The level curve defined by $f(x,y) = 1$ has a tangent line parallel to $\langle 3, 1\rangle$ at the point $(1,-1)$

True

(1) If C_1 and C_2 intersect



then (x_1, y_1, z_1) (x, y, z_2) are both on the surface. Can't happen if f is a function

(2) Thm in the book

(3) $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} \\ 0 \quad (x,y)=(0,0) \end{cases}$

(4) $-\nabla f$

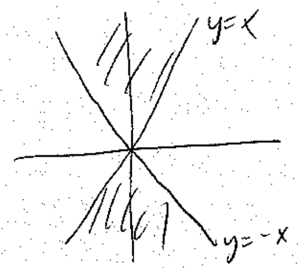
(5) $\nabla f(1,-1) = \langle -2x - 3x^2 y, \ 2y - x^3\rangle_{(1,-1)} = \langle -2+3, -2-1\rangle = \langle 1, -3\rangle$

is $\perp$ to level curve so tangent line is $\perp$ to $\langle 1,-3\rangle$ and $\langle 3, 1\rangle$

$z=$

Problem 2 (10 points): Consider the function $f(x, y) = \sqrt{y^2 - x^2}$.

(1) Sketch the domain of f and describe the range of f.



3

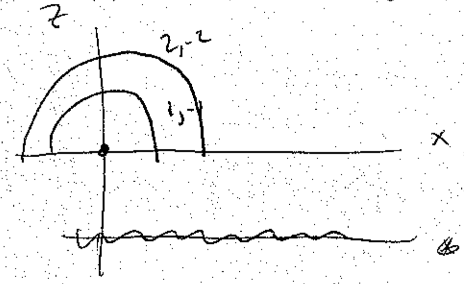
algebra (+1)

range(f): $\{z \geq 0\}$

2

both halves (−2)

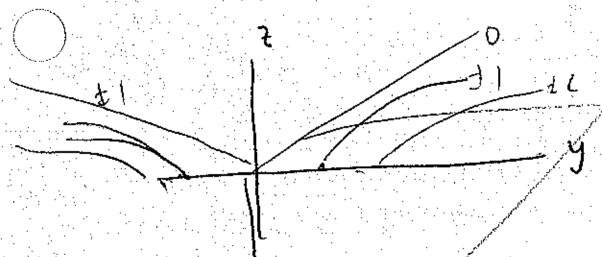
(2) Draw the slices for $y = -2, -1, 0, 1, 2$.



2

$y = \sqrt{1-x^2}$

(3) Draw the slices for $x = -2, -1, 0, 1, 2$.



$z = \sqrt{y^2 - 1}$

$\Leftrightarrow$ top half of

$1 = y^2 - z^2$

$4 = y^2 - z^2$

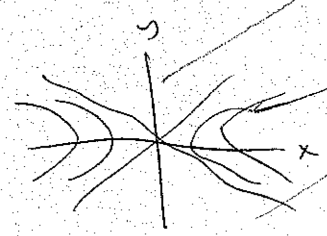
(4) Draw the slices for $z = -2, -1, 0, 1, 2$.

$0 = \sqrt{y^2 - x^2} \Rightarrow y = \pm x$

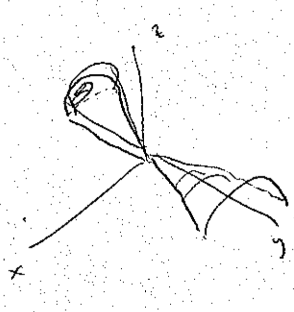
$1 = y^2 - x^2$

-1 nothing

-2 nothing



(5) Describe (or sketch) the graph of $z = f(x, y)$.



3

units

Problem 3 (10 points): Some word problems:

(1) Suppose W represents average annual wheat production in bushels/acre and that W is a differentiable function of the variables T (average annual temperature in °C) and R (annual rainfall in cm). Also suppose that $\partial W/\partial T = -2$bu./° C and $\partial W/\partial R = 8$bu./cm. If, in some 5 year period, T increases at a rate of 0.15° C/yr and R decreases at a rate of 0.1cm/yr. By how many bushels per acre does the value of W decrease over this five year period?

$W(T,R)$

$$\frac{\partial W}{\partial t} = \frac{\partial W}{\partial T}\frac{dT}{dt} + \frac{\partial W}{\partial R}\frac{\partial R}{\partial t} \quad \times 2$$

$$= -2(0.15) + 8(-.1)$$

$$= -.3 - .8 = -1.1 \text{ bu/acre/yr.} \quad \times 1$$

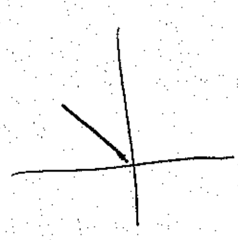
Over five years $(-1.1)5 = -5.5$ $\times 1$ **5**

It decreases by 5.5 bu/acre/yr $\times 1$

(2) Suppose that the temperature in °C at the point (x, y) is given by $T(x, y) = 10 - (0.003)x^2 + (0.004)y^2$. If you are located at the point $(4, 3)$ and are feeling cold, in which direction should you move to get warmer most quickly? Describe the direction as an angle from the positive x-axis moving in the counterclockwise direction.

$$\nabla T = \langle -0.006x, 0.008y\rangle\big|_{(4,3)} \quad \times 4$$ **5**

$$= \langle -0.024, 0.0024\rangle$$



$\frac{3\pi}{4}$ $\times 1$

Problem 4 (10 points): Consider the function $f(x,y) = x\ln(x^3+y)$.

(1) Find the linear approximation $L(x,y)$ for $f(x,y)$ which is valid for all (x,y) near the point $(1,0)$. Use this formula to approximate $f(1.1,-0.1)$.

$$L(x,y) \simeq f(1,0) + f_x(1,0)(x-1) + f_y(1,0)(y-0)$$

$$f_x = \ln(x^3+y) + \frac{x \cdot 3x^2}{x^3+y}, \quad f_x(1,0) = \ln(1) + \frac{3}{1} = 3$$

$$f_y = \frac{x}{x^3+y}, \quad f_y(1,0) = 1$$

$$L(x,y) = 0 + 3(x-1) + y = 3x+y-3.$$

$$f(1.1,-0.1) \approx 3.3 - 0.1 - 3 = \boxed{0.2}$$

(2) What is the equation of tangent plane for $z = f(x,y)$ at the point $(1,0,0)$?

$$\boxed{z = 3x+y-3}$$

Calculate

(3) Calculus $\partial^2 f/\partial x^2$ and $\partial^2 f/\partial x \partial y$ for any (x,y).

$$f_{xx} = \frac{3x^2}{x^3+y} + \frac{(x^3+y)(9x^2) - 3x^3(3x^2)}{(x^3+y)^2}$$

$$f_{xy} = f_{yx} = \frac{(x^3+y) - x(3x^2)}{(x^3+y)^2}.$$

Problem 5 (6 points): Describe the following surfaces (give coordinates of centers, vertices, etc where appropriate).

(1) $\rho = 2\sin\phi\sin\theta$

$\sqrt{x^2+y^2+z^2} = \frac{2y}{\rho} \Rightarrow x^2+y^2+z^2 = 2y$

$\Rightarrow x^2 + y^2 - 2y + z^2 = 0$

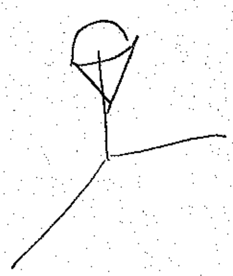
$\Rightarrow x^2 + (y-1)^2 + z^2 = 1$

sphere of radius 1, centered at $(0, 1, 0)$

(2) $z = 1 + r$.

$z = 1 + \sqrt{x^2+y^2}$

Cone shifted up one, vertex at $(0,0,1)$
circular, right.



OR

$z = 0 \Rightarrow r = -1$ nothing

$z = 1 \Rightarrow r = 0 \Rightarrow (0,0,1)$

$z = 2 \Rightarrow r = 1$

$z = 3 \Rightarrow r = 2$

Problem 6 (9 points): Consider the function

$$f(x,y,z) = x^2 + 2y^2 + z^2/2 + 2xy$$

whose level surfaces are all ellipsoids.

(1) Find the directional derivative at $(-1,1,2)$ in the direction of $3\mathbf{i} + 4\mathbf{k}$.

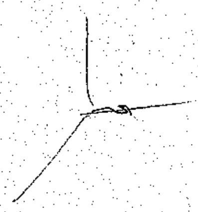
$D_u f(P) = \nabla f_p \cdot \vec{u}$, here $\vec{u} = \langle \frac{3}{5}, 0, \frac{4}{5} \rangle$ +✓

$\nabla f_p = \langle 2x+2y, 4y+2x, z \rangle_{(-1,1,2)} = \langle 0, 2, 2 \rangle$ +2

$D_u f(p) = \langle 0,2,2 \rangle \cdot \langle \frac{3}{5}, 0, \frac{4}{5} \rangle = \boxed{\frac{8}{5}}$

+2

(2) Find all the points on the level surface $f(x,y,z) = 4$ where the tangent plane to the surface is parallel to the xz-plane.



~~Norm~~ These planes have normal vector parallel to j, $\langle 0,1,0 \rangle$.

The gradient is normal to level surfaces.

+3

$4 = x^2 + 2y^2 + \frac{z^2}{2} + 2xy$

$\nabla f = \langle 2x+2y, 4y+2x, z \rangle$

$z = 0$

$2x + 2y = 0$

$4y + 2x = \lambda$

$4y - 2y = \lambda$

$2y = \lambda$

$y = \frac{\lambda}{2}$

$z = 0$

$x = -\frac{\lambda}{2}$.

$\frac{\lambda^2}{4} + 2 \cdot \frac{\lambda^2}{4} + 0 - \frac{2\lambda^2}{4} = 4$

$\lambda = \pm 4$

$\boxed{\begin{array}{l} y = \pm 2 \\ x = \mp 2. \\ z = 0. \end{array}}$