Math 211

February 9, 2012
First Midterm

NAME: ______________________ 1pm or 2pm__________

Problem	Points	Score
1	5	
2	10	
3	10	
4	9	
5	9	
6	7	
Total	50	

Problem 1 (5 points): Answer the following True and False questions. You will get 1 point for a correct answer, lose 1 point for a wrong answer and will neither lose nor get a point for leaving it blank. Write out the entire word: TRUE or FALSE

(1) The area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$ is the same as the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{v} + \mathbf{w}$.

True

(2) The angle between the lines

$$\mathbf{r}_1(t) = \langle 1 + 3t, -1, 4 - 3t \rangle$$
$$\mathbf{r}_2(t) = \langle 1 + t, 1 - t, 2 \rangle$$

is $\frac{\pi}{4}$.

False

(3) Consider the vector field $\mathbf{F} = \langle 0, x \rangle$ and a particle moving along the x-axis. Then there is no work done on the particle by $\mathbf{F}$.

True

(4) Suppose the trajectories of two particles are given by

$$\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$$
$$\mathbf{r}_2(t) = \langle 2t - 4, 3t + 4, 2t^2 + 2t + 24 \rangle.$$

Then the two particles will collide. @ time t=4

True

(5) If $\mathbf{u} \cdot \mathbf{v} = 0$ and $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

True

(1) $\|(v+w) \times w\| = \|v \times w\|$

(2) $\langle 3, 0, -3 \rangle \cdot \langle 1, -1, 0 \rangle = 3 = \|\sqrt{18}\ \sqrt{2}\ \cos\theta$

$\frac{1}{2} = \cos\theta \Rightarrow \theta = \frac{\pi}{3}$

(3) $\int_a^b \langle 0, t \rangle \cdot \langle 1, 0 \rangle \, dt = 0.$

(4) $t = 2t - 4$ $t = 4.$

$t^2 = 3t + 4$ @4 $\Rightarrow 16 = 16$

$64 = 2 \cdot 16 + 8 + 24$ ✓

(5) $u \cdot v = 0 \Rightarrow$ $u \perp v$ or one of them is 0. $u \times v = 0 \Rightarrow \|u\| \|v\| \sin\theta = 0 \Rightarrow$

$u \parallel v$ or one of them is zero

Problem 4 (9 points): Consider the position function $\mathbf{r}(t) = \langle t, t^2, 3t \rangle$. Find the normal and tangential components of the particle's acceleration.

$$v(t) = r'(t) = \langle 1, 2t, 3 \rangle$$

3

$\cancel{N(t)} =$

$$a(t) = r''(t) = \langle 0, 2, 0 \rangle$$

$$a_T = \frac{\vec{a} \cdot \vec{v}}{\|\vec{v}\|} = \frac{\langle 0,2,0 \rangle \cdot \langle 1, 2t, 3 \rangle}{\sqrt{10 + 4t^2}}$$

3

$$= \frac{4t}{\sqrt{10+4t^2}}$$

$$a_N = \sqrt{4 - \frac{16t^2}{10+4t^2}}$$

3

$$= \sqrt{\frac{4(10+4t^2) - 4t^2}{10+4t^2}}$$

$$= \sqrt{\frac{40}{10+4t^2}}$$

Problem 5 (9 points): Let C be the arc of the parabola $y = 2x^2$ from $(1, 2)$ to $(-1, 2)$. Let $\mathbf{F}$ be the vector field $\langle 3x^2, 3y^2 \rangle$. Find

$$\int_C \mathbf{F} \cdot ds.$$

C: $c(t) = \langle t, 2t^2 \rangle \quad -1 \leq t \leq 1$

$$-\int_{-1}^{1} F(t, 2t^2) \cdot \langle 1, 4t \rangle \, dt$$

$$= -\left(\int_{-1}^{1} \langle 3t^2, 12t^4 \rangle \cdot \langle 1, 4t \rangle \, dt\right)$$

$$= -\left(\int_{-1}^{1} 3t^2 + 48t^5 \, dt\right)$$

$$= -\left(t^3 + 8t^6 \Big|_{-1}^{1}\right)$$

$$= -\left(1 + 8 - (-1 + 8)\right)$$

$$\boxed{= -2}$$

Problem 2 (10 points): Two vector questions:

(1) Find a unit vector orthogonal to both $\langle 1, -1, 1\rangle$ and $\langle 0, 2, 2\rangle$.

$$\langle 1,-1,1\rangle \times \langle 0,2,2\rangle = \det \begin{pmatrix} i & j & k \\ 1 & -1 & 1 \\ 0 & 2 & 2 \end{pmatrix}$$

$$= i \det\begin{pmatrix} -1 & 1 \\ 2 & 2 \end{pmatrix} - j \det\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} + k \det\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$$

$$= i(-4) - 2j + 2k$$

$$= \langle -4, -2, 2\rangle$$

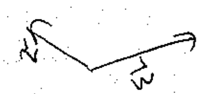
unit vector:

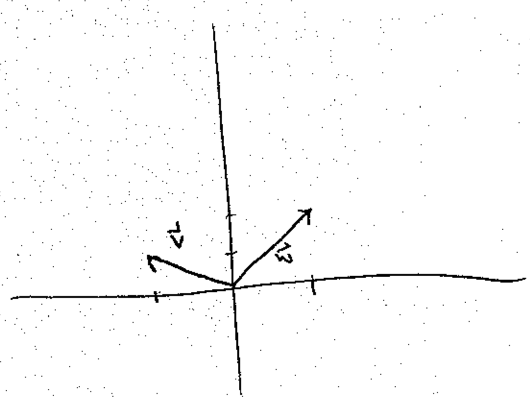
$$\frac{1}{\sqrt{4^2+2^2+2^2}} \langle -4,-2,2\rangle = \frac{1}{\sqrt{24}} \langle -4,-2,2\rangle$$

(2) Sketch two vectors $\mathbf{v}, \mathbf{w}$ in the xy-plane such that $\mathbf{v} \cdot \mathbf{w} < 0$, $\mathbf{v} \times \mathbf{w}$ is in the same direction as $-\mathbf{k}$, the x-component of $\mathbf{v} + \mathbf{w}$ is 0 and the y-component of $2\mathbf{v} - \mathbf{w}$ is 0.

$v \cdot w < 0$ means obtuse angle

$v \times w \parallel -k \Rightarrow$





Problem 3 (10 points): Two questions are parameterizations:

(1) Find the equation of the line that is tangent to the curve

$$\mathbf{r}(t) = \langle t, t^2, \tfrac{2}{t} \rangle$$

when $t = 2$.

$r'(t) = \langle 1, 2t, -\frac{2}{t^2} \rangle$ 2

$r'(2) = \langle 1, 4, -\frac{1}{2} \rangle$ 2

$\ell(t) = \langle 2, 4, 1 \rangle + t \langle 1, 4, -\frac{1}{2} \rangle$ 1

7 (2) Describe, in words, the curve traced out by

$$\mathbf{r}(t) = \langle 19 - 4\sin 3t, 23 + 3\cos\tfrac{t}{3} \rangle. \quad -\infty < t < \infty$$

an ellipse in the xy-plane centered at $(19, 23)$

minor axis in the y-direction of length 3

major axis in the x-direction of length 4.

Problem 6 (7 points): A particle travels one time around the helix

$$\mathbf{r}(t) = \langle \cos 2t, \sin 2t, t \rangle.$$

Find the total distance traveled by the particle.

To go one time around the helix means going from $0 \le t \le \pi$.

So

$$\int_0^\pi \|r'(t)\|\,dt = \int_0^\pi \sqrt{\cancel{2\cos 2t}\ (-2\sin 2t)^2 + (2\cos 2t)^2 + 1}\ dt$$

$$= \int_0^\pi \sqrt{4\sin^2 2t + 4\cos^2 2t + 1}\ dt$$

$$= \int_0^\pi \sqrt{5}\ dt$$

$$= \sqrt{5}\,\pi.$$

12.1 37, 39, 41

12.2 ~~47~~, 39, 49, 51

12.3 47, 69, 71, 81, 82

12.4 ~~8~~ 41, 53

13.1 10, 29,

13.2 17, 23, 31, 39, 47

13.3 15, 25,

13.5 19, 33, 52

16.1 Prelim Qs, 4, 17-20, 24.

~~16.1~~

16.2 Prelim Qs 13, 17, 22, 28