Integrals and Integration Theorems Review

Problem 1 For these problems find $\int_C \vec{F} \cdot d\vec{s}$ where $\vec{F}$ and C are given:

(1) $\vec{F} = \langle x, y, z \rangle$ and C is the line segment from $(0,0,0)$ to $(1,1,1)$.
(2) $\vec{F} = \langle x, y, z \rangle$ and C is parameterized by $\vec{c}(t) = \langle t, \sqrt{t}, \sqrt[3]{t} \rangle$ $(t \in [0,1])$.
(3) $\vec{F} = \langle 2xy, x^2 + z, y + 2z \rangle$ and C is parameterized by $\vec{c}(t) = \langle t^2 - t, \sin(\pi t), \cos^2(\pi t) \rangle$ $(t \in [0,1])$.
(4) $\vec{F} = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$ and C is the unit circle in the xy-plane oriented counterclockwise.
(5) $\vec{F} = \langle \sqrt{x} + y^3, x^2 + \sqrt{y} \rangle$ and C consists of the sine curve from $(0,0)$ to $(\pi, 0)$.

Problem 2 For these problems find $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F}$ and S are given:

(1) $\vec{F} = \text{curl}\langle 2y, z - 2x, yz \rangle$ and S is the hemisphere of radius 1, centered at the origin, above the xy-plane, oriented with the upward pointing normal.
(2) $\vec{F} = \text{curl}\langle 2y, z - 2x, yz \rangle$ and S is the disk of radius 1, centered at the origin, in the xy-plane, oriented with the upward pointing normals.
(3) $\vec{F} = \text{curl}\langle 2y, z - 2x, yz \rangle$ and $S = S_1 \cup S_2$ is the union of the surfaces from the previous two problems, oriented with the outward pointing normals.
(4) $\vec{F} = \text{curl}\langle 2z, 3x, 5y \rangle$ and S is the surface given parametrically by $G(r, \theta) = \langle r\cos\theta, r\sin\theta, 4 - r^2 \rangle$ for $(r, \theta) \in [0,3] \times [0, 2\pi]$.
(5) $\vec{F} = \langle x^2 y - 3x, -xy^2 + 2\cos(y)z, \sin(y)z^2 \rangle$ and $S = S_1 \cup S_2$ as in two problems ago.
(6) $\vec{F} = \langle x, y, z^4 \rangle$ and S is the part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$, oriented downward.

Problem 3 For these problems, find $\iiint_E \text{div}\vec{F}\ dV$ where $\vec{F}$ and E are given:

(1) $\vec{F} = \text{curl}\vec{G}$ where $\vec{G}$ is any appropriately smooth vector field and E is any simple solid.
(2) $\vec{F} = \langle x, y^2, -2yz \rangle$ and E is the solid ball of radius a centered at the origin.
(3) $\vec{F} = \langle x^2, 2yz, x^2 - z^2 \rangle$ and E is the unit cube with corners at $(0,0,0)$ and $(1,1,1)$.