(1) use stokes theorem

$\partial S = \langle \cos t, \sin t, 0 \rangle$

$\iint_S \nabla \times F \cdot d\vec{S} = \oint_{\partial S} \langle 2\sin t, 0 - 2\cos t, 0 \rangle \cdot \langle -\sin t, \cos t, 0 \rangle \, dt$

$= \int_0^{2\pi} -2\sin^2 t - 2\cos^2 t \, dt$

$= -4\pi.$

(2) we're done; same as (1) since and have the same boundary.

(3) Divergence theorem:

$\iint_S \text{curl} \langle 2y, z - 2x, yz \rangle \cdot d\vec{S} = \iiint \text{div curl} \langle \, \rangle \, dV$

$= 0.$

(4) Need to find the orientation: $(G_r \times G_\theta) \Rightarrow$ ~~view~~ $\langle 2r^2\cos\theta, 2r\sin^2\theta, r \rangle$

So upward pointing $\Rightarrow$ C has counterclockwise orientation..

$c(t) = \langle 3\cos t, 3\sin t, -5 \rangle$

$\iint_S \text{curl}(F) \cdot d\vec{S} = \oint_C \langle -10, 9\cos t, 15\sin t \rangle \cdot \langle -3\sin t, 3\cos t, 0 \rangle dt$

$= 27\pi.$

(5) Divergence Theorem.

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F} \, dV$$

$$= \iiint_E 2xy - 3 - 2xy - 2\sin yz + 2z\sin y \, dV$$

$$= \iiint_E -3 \, dV$$

$$= -3\left(\frac{4}{3}\pi (1)^3\right)\frac{1}{2} = -2\pi.$$

(6) Directly: (or divergence):

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_D -F_1 f_x \mp F_2 f_y + F_3 \, dA$$

$$= \iint_D \frac{-x^2}{\sqrt{x^2+y^2}} - \frac{y^2}{\sqrt{x^2+y^2}} + z^4 \, dA$$

$$= \iint_D -\sqrt{x^2+y^2} + (x^2+y^2)^2 \, dA$$

$$= \int_0^{2\pi}\int_0^1 (r^4 - r)\, r \, dr \, d\theta = -\frac{\pi}{3}$$

(1) $\int_c \vec{F} \cdot d\vec{s}$ $\qquad c(t) = \langle t, t, t \rangle \quad 0 \le t \le 1$

$$\int_0^1 \langle t, t, t \rangle \cdot \langle 1, 1, 1 \rangle \, dt = \int_0^1 3t = \frac{3}{2}.$$

(2) $\int_C \vec{F} \cdot d\vec{s}$ $\qquad c(t) = \langle t, \sqrt{t}, \sqrt[3]{t} \rangle$
$0 \le t \le 1$. a little grosser.

Is $\vec{F}$ conservative?

$$f = \frac{1}{2}(x^2 + y^2 + z^2) \qquad f(1,1,1) - f(0,0,0) = \frac{3}{2}.$$

Yes. So; I can do ~~since~~ such an $\vec{F}$ is path independent

(3) Is $\vec{F}$ conservative?

$$\frac{\partial f}{\partial x} = 2xy \Rightarrow f(x,y,z) = x^2y + g(y,z)$$

$$\frac{\partial f}{\partial y} = x^2 + \frac{\partial g}{\partial y} \Rightarrow \frac{\partial g}{\partial y} = z$$

$$\Rightarrow g(y,z) = yz + h(z)$$

$$f(x,y,z) = x^2y + yz + h(z)$$

$$\frac{\partial f}{\partial z} = y + h'(z) \Rightarrow h(z) = z^2.$$

So $f(x,y,z) = x^2y + yz + z^2$

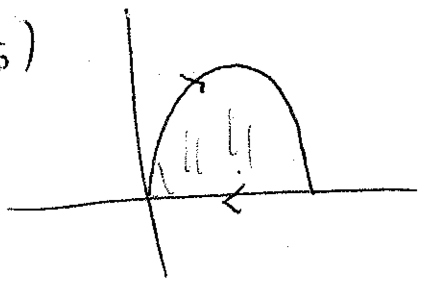
The path is closed, so $\int_C \vec{F} \cdot d\vec{s} = 0.$

(4) ① $\frac{\partial f_1}{\partial y} = \frac{\partial f_2}{\partial x}$ so $\vec{F}$ conservative so $\oint \vec{F}\cdot d\vec{s} = 0$. No!. worksheet

② Green's Thm: $\iint_D \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\, dA = 0$ No!. GT requires continuous partial in the region

Directly $\int_0^{2\pi} \left\langle \frac{-\sin t}{1}, \frac{\cos t}{1} \right\rangle \langle -\sin t, \cos t\rangle\, dt = \int_0^{2\pi} \sin^2 t + \cos^2 t\, dt = 2\pi.$

(5)



Conservative? $\frac{\partial F_1}{\partial y} = 3y^2 \neq \frac{\partial F_2}{\partial x} = 2x$ So no.

Directly? $F(t)c'(t) = \langle \sqrt{t} + \sin^3 t,\ t^2 + \sqrt{\sin t} \rangle \cdot \langle 1, \cos t\rangle$

$c(t) = (t, \sin t)$
$c'(t) = (1, \cos t)$

OK, but gross.

Green's Theorem:
Close it up w/ C': $\langle 0, t\rangle$ $0 \le t \le \pi$

$\oint_{\partial D} \vec{F}\cdot d\vec{s} = \int_C \vec{F}\cdot d\vec{s} + \int_{C'} \vec{F}\cdot d\vec{s}$

$= \iint_D \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\, dA$

$= \iint_D 2x - 3y^2\, dA$

$= \int_0^{\pi}\int_0^{\sin x} 2x - 3y^2\, dy\, dx = 2\pi - \frac{4}{3}.$

$\int_{C'} \vec{F}\cdot d\vec{s} = -\int_{C'} \vec{F}\cdot d\vec{s}$

$= -\int_0^{\pi} \langle \sqrt{t}, t^2\rangle \cdot \langle 1, 0\rangle\, dt$

$= -\frac{2}{3}\pi^{3/2}.$

So $\int_C = \iint_D - \int_{C'} = 2\pi - \frac{4}{3} + \frac{2}{3}\pi^{3/2}$

(1) $\iiint_E \operatorname{div} \vec{F} \, dV = \iiint_E \operatorname{div} (\operatorname{curl} (\vec{F})) \, dV = \iiint 0 \, dV = 0$

(2) $\iiint_E \operatorname{div} \vec{F} = \iiint_E 1 + 2y - 2y \, dV = \frac{4}{3} \pi a^3$

(3) $\iiint_E \operatorname{div} \vec{F} \, dV = \iiint_E 2x \, dV = \int_0^1 \int_0^1 \int_0^1 2x \, dx \, dy \, dz = 1$