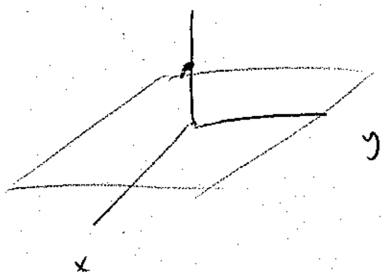
12.2   29, 37, 39, 41, 47, 49, 51, 53

2 $r(t) = \langle 1,2,-8\rangle + t\langle 2,1,3\rangle$

$= (1+2t,\ 2+t,\ -8+3t)$

37



$r(t) = \underbrace{\langle 0,0,0\rangle}_{\text{point}} + t\underbrace{\langle 0,0,1\rangle}_{\text{direction vector}}$

$= \langle 0,0,t\rangle$

39 I need the direction vector and a point.

$\langle 1,1,0\rangle - \langle 0,-1,-2\rangle = \langle 1,2,2\rangle.$

So

$r(t) = \langle 0,0,4\rangle + t\langle 1,2,2\rangle.$

41 Direction vector is $\langle 0,1,0\rangle$ so, (C).

47 Two lines are the same if they're parallel and contain the same point.

r_1 and r_2 are parallel since ~~their~~ they have parallel direction vectors:

$\underbrace{\langle 8,12,-6\rangle}_{\text{direction of } r_1} = 2\underbrace{\langle 4,6,-3\rangle}_{\text{direction of } r_2}$

r_1 clearly contains $(3,-1,4)$ (at $t=0$), does r_2? Can I find a t so that

$(3,-1,4) = (11+4t,\ 11+6t,\ -2-3t)$? $11+4t = 3 \Rightarrow t=-2.$

$11+6(-2) = -1$ ✓

$-2-3(-2) = 4$ ✓.

So r_2 contains $(3,-1,4)$ @ $t=-2$

49

$r_1(t) = \langle 5,5,2\rangle + t\langle 0,-2,1\rangle$

$r_2(t) = \langle 5,5,2\rangle - t\langle 0,-2,1\rangle$ (diff direction vector)

$r_3(t) = \langle 5,5,2\rangle + \langle 0,-2,1\rangle + t\langle 0,-2,1\rangle$

$= \langle 5,3,3\rangle + t\langle 0,-2,1\rangle$ (diff initial point)

53 ~~If two lines intersect, there is some t_1 and some t_2 so that~~

$r_1(t_1) = r_2(t_2)$.

So,

$\langle 1,0,0\rangle + t_1\langle -3,1,0\rangle = \langle 0,1,1\rangle + t_2\langle 2,0,1\rangle$.

and

$$\left.\begin{array}{l} 1-3t_1 = 2t_2 \\ t_1 = 1 \\ 0 = 1+t_2 \end{array}\right\} \Rightarrow \begin{array}{l} t_1 = 1 \\ t_2 = -1 \end{array}$$

Plugging into 1st equation we get

53

I have no idea what problem I just did

51 Show $\langle -1,2,2\rangle + t\langle 4,-2,1\rangle$

and $\langle 0,1,1\rangle + t\langle 2,0,1\rangle$ don't intersect.

i.e., show there is no t_1 and t_2 so that

$$\langle -1,2,2\rangle + t_1\langle 4,-2,1\rangle = \langle 0,1,1\rangle + t_2\langle 2,0,1\rangle.$$

i.e.,

$$\left.\begin{array}{l} -1 + 4t_1 = 2t_2 \\ 2 \cancel{+} -2t_1 = 1 \Rightarrow t_1 = \frac{1}{2} \\ 2 + t_1 = 1 + t_2 \end{array}\right\} \Rightarrow t_2 = \frac{3}{2}$$

Plug $t_1 = \frac{1}{2}$, $t_2 = \frac{3}{2}$ in the first equation.

$-1 + 2 \neq 2 \cdot \frac{3}{2}$. So there is no such t_1, t_2 so

the lines don't intersect.

Do $\langle 0,1,1\rangle + t\langle 1,1,2\rangle$

and $\langle 2,0,3\rangle + s\langle 1,4,4\rangle$ intersect?

I.e., can I solve

$$\left.\begin{aligned} t &= 2+s \\ 1+t &= 4s \end{aligned}\right\} \Rightarrow 3+s=4s \Rightarrow s=1 \Big\} \Rightarrow t=3.$$

$$1+2t = 3+4s$$

$$1+2(3) = 3+4(1) \checkmark.$$

So a point in common is the point head of

$$\langle 0,1,1\rangle + 3\langle 1,1,2\rangle = \langle 3,4,7\rangle.$$

So $\boxed{(3,4,7)}$.