Some useful facts:

Basics: $n! = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1 = n \cdot (n-1)!$, $0! = 1$; $\binom{n}{k} = \frac{n!}{k!(n-k)!}$; $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$; $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$.

Combinatorial interpretations: $n!$ is number of ways of rearranging n objects where order matters. $\binom{n}{k}$ is the number of k-element subsets that can be picked from a set of n elements.

Combinatorial proofs: Consider the identity $\binom{n}{n-k} = \binom{n}{k}$. Algebraic proof is pretty easy. Combinatorial proof: show that the right hand side and the left side count the same things. The LHS counts the number of ways to count sets with k elements. The RHS counts the numbers of ways to exclude k elements. These numbers are the same.

Another combinatorial proof: Consider

$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$$

rewritten as

$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

The LHS is the number of ways to pick a board of directors from n people and then choose a chairperson from within that board. That's the name as choosing a chairperson from n people and then making up the rest of the $k-1$ boardmembers from the remaining $n-1$ people.