

### HOMEWORK 8 – NUMBER THEORY I

Remember, you need to work on these for an hour and a half and you need to show me some evidence that you did. Try small cases. Plug in smaller numbers. Do examples. Look for patterns. Draw pictures. Use lots of paper. Choose effective notation. Look for symmetry. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Don't be afraid of a little algebra.

- 1:** Let  $f$  be a polynomial with positive integer coefficients. Prove that if  $n$  is a positive integer, then  $f(n)$  divides  $f(f(n) + 1)$  if and only if  $n = 1$ .
- 2:** Let  $A_1 = 0$  and  $A_2 = 1$ . For  $n > 2$ , the number  $A_n$  is defined by concatenating the decimal expansions of  $A_{n-1}$  and  $A_{n-2}$  from left to right. For example,  $A_3 = A_2A_1 = 10$ ,  $A_4 = A_3A_2 = 101$ , etc. Determine (with proof) all  $n$  such that 11 divides  $A_n$ .
- 3:** How many primes are there whose base 10 expansions consist of alternating 1's and 0's?