Homework 7 – Integrals

Remember, you need to work on these for an hour and a half and you need to show me some evidence that you did. Try small cases. Plug in smaller numbers. Do examples. Look for patterns. Draw pictures. Use lots of paper. Choose effective notation. Look for symmetry. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Don't be afraid of a little algebra.

1: Evaluate

$$\int_0^1 \frac{\ln(x+1)}{x^2+1}\,dx.$$

2: Suppose that $f : [0,1] \to \mathbb{R}$ has a continuous derivative and that $\int_0^1 f(x)\,dx = 0$. Prove that for every $\alpha \in (0,1)$,

$$\left|\int_0^\alpha f(x)\,dx\right| \leq \frac{1}{8} \max_{0\leq x\leq 1} |f'(x)|.$$

3: Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x & \text{if } x \leq e \\ xf(\ln(x)) & \text{if } x > e. \end{cases}$$

Does the series

$$\sum_{n=1}^{\infty} \frac{1}{f(n)}$$

converge?