

HOMEWORK 6 – DERIVATIVES

Remember, you need to work on these for an hour and a half and you need to show me some evidence that you did. Try small cases. Plug in smaller numbers. Do examples. Look for patterns. Draw pictures. Use lots of paper. Choose effective notation. Look for symmetry. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Don't be afraid of a little algebra.

1: Let k be a fixed positive integer. The n -th derivative of $\frac{1}{x^k-1}$ has the form $\frac{P_n(x)}{(x^k-1)^{n+1}}$ where $P_n(x)$ is a polynomial. Find $P(1)$.

2: Find all real-valued continuously differentiable functions f on the real line such that for all x

$$(f(x))^2 = \int_0^x [(f(t))^2 + (f'(t))^2] dt + 1990.$$

3: Find all the values of α for which the curves

$$y = \alpha x^2 + \alpha x + \frac{1}{24}$$

$$x = \alpha y^2 + \alpha y + \frac{1}{24}$$

are tangent to each other.