Homework 5 – Sequences and Series

Remember, you need to work on these for an hour and a half and you need to show me some evidence that you did. Try small cases. Plug in smaller numbers. Do examples. Look for patterns. Draw pictures. Use lots of paper. Choose effective notation. Look for symmetry. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Don't be afraid of a little algebra.

0: Consider the recurrence given by $a_n = 6a_{n-1} - 7a_{n-2}$, $a_0 = 3$, $a_1 = 8$. Find an explicit formula for a_n.

1: A subset S of $\{1, 2, \ldots, n\}$ is **self-centered** if the cardinality of S is an element of S. Find the number of proper self-centered sets, i.e., the number self-centered subsets of $\{1, \ldots, n\}$ that do not have another self-centered proper subset.

2: For each integer $n \geq 0$, let $S(n) = n - m^2$ where m is the greatest integer so that $m^2 \leq n$. Define a sequence $(a_k)_{k=0}^{\infty}$ by $a_0 = A$ and $a_{k+1} = a_k + S(a_k)$ for $k \geq 0$. For what positive integers A is this sequence eventually constant? Be sure to prove that no number outside your set of positive integers has this property, as well.

3: Suppose that a sequence $a_1, a_2, a_3, \ldots$ satisfies $0 < a_n \leq a_{2n} + a_{2n+1}$ for all $n \geq 1$. Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.