Homework 4 – Linear Algebra

Remember, you need to work on these for an hour and a half and you need to show me some evidence that you did. Try small cases. Plug in smaller numbers. Do examples. Look for patterns. Draw pictures. Use lots of paper. Choose effective notation. Look for symmetry. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Dont be afraid of a little algebra.

1: In determinant tic-tac-toe, Player A enters a one anywhere in a 3 by 3 matrix. Player B enters a zero in any vacant spot. Play continues until the matrix is filled with 5 ones and 4 zeros. Player B wins if the determinant of the matrix is 0, Player A wins if the determinant of the matrix is not zero. Assuming both players puruse optimal strategies, who will win and how?

2: Let d_n be the determinant of the $n \times n$ matrix whose entries from left to right and from top to bottom are $\cos 1,\ \cos 2, \ldots, \cos n^2$. Evaluate $\lim_{n\to\infty} d_n$.

3: Do there exist polynomials $a(x), b(x), c(y), d(y)$ such that $1 + xy + x^2y^2 = a(x)c(y) + b(x)d(y)$?