## Homework 3 – Polynomials

Remember, you need to work on these for an hour and a half and you need to show me some evidence that you did. Try small cases. Plug in smaller numbers. Do examples. Look for patterns. Draw pictures. Use lots of paper. Choose effective notation. Look for symmetry. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Dont be afraid of a little algebra.

**1:** Find all ordered triples that satisfy

$$\begin{aligned} x + y - z &= 0 \\ xz - xy + yz &= 27 \\ xyz &= 54 \end{aligned}$$

**2:** The roots of $x^3 + ax^2 + bx + c = 0$ are $\alpha, \beta, \gamma$. Find the cubic whose roots are $\alpha^3, \beta^3, \gamma^3$.

**3:** Let $p(z)$ be a polynomial of degree $n$ all of whose zeros have absolute value 1 in the complex plane. Put $g(z) = \frac{p(z)}{z^{n/2}}$. Show that all the zeros of $g'(z)$ have absolute value 1 (ignore $z = 0$ and choose the obvious branch of the square root of $z$).