Homework 2 - Algebraic Equalities and Some Inequalities

Remember, you need to work on these for an hour and a half and you need to show me some evidence that you did. Try small cases. Plug in smaller numbers. Do examples. Look for patterns. Draw pictures. Use lots of paper. Choose effective notation. Look for symmetry. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Dont be afraid of a little algebra.

1: Prove that

$$\frac{5^{125}-1}{5^{25}-1}$$

is not prime. This can be done in a way similar to a problem we did in class.

2: Find all positive integers $n, k_1, \ldots, k_n$ so that

$$k_1 + k_2 + \cdots + k_n = 5n - 4$$

and $\frac{1}{k_1} + \cdots + \frac{1}{k_n} = 1$.

3: Find all real solutions to

$$\frac{4x^2}{4x^2+1} = y$$
$$\frac{4y^2}{4y^2+1} = z$$
$$\frac{4z^2}{4z^2+1} = x$$