

HOMEWORK 1 - BASIC PROOF TECHNIQUES

Do one of each kind of problem: appetizers should be rather straightforward applications of the ideas covered in class; entrees should be less straightforward and desserts should be hard. Good luck and have fun!

Remember, you need to work on these for an hour and you need to show me some evidence that you did. Try small cases. Plug in smaller numbers. Do examples. Look for patterns. Draw pictures. Use lots of paper. Choose effective notation. Look for symmetry. Divide into cases. Work backwards. Argue by contradiction. Consider extreme cases. Modify the problem. Generalize. Don't be afraid of a little algebra.

- 1:** In class I gave a sketch of proof of the following statement: Let $n \geq 6$ be an integer. Show that

$$\left(\frac{n}{3}\right)^n < n! < \left(\frac{n}{2}\right)^n .$$

Finish the details of this proof.

- 2:** Show that the only functions $f : \mathbb{N} \rightarrow \mathbb{N}$ satisfying

$$xf(y) + yf(x) = (x + y)f(x^2 + y^2)$$

are the constant functions.

- 3:** In each of the unit squares of a 10 by 10 checkerboard, a positive integer not exceeding 10 is written. Any two numbers that appear in adjacent or diagonally adjacent squares of the board are relatively prime. Prove that some number appears at least 17 times.