Section 6.5

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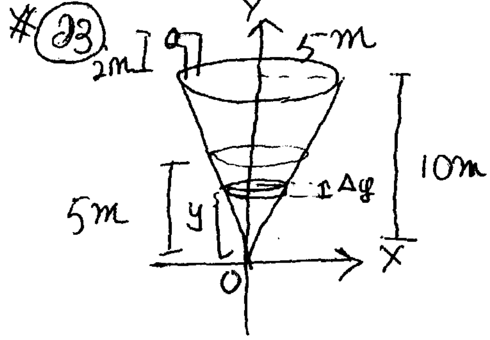
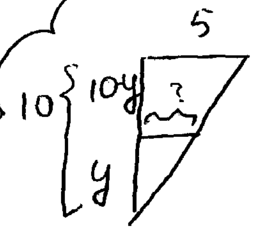


Volume of the slice

$$= \pi \left(\frac{y}{2}\right)^2 \Delta y$$



$$\frac{5}{10} = \frac{?}{y}$$

$$? = y/2$$

Work done to empty the slice of water

= weight of the slice × distance for the slice of water to travel

= mg × distance for the slice of water to travel

= (density of water × volume of the slice) × g × distance (∵ density = $\frac{\text{mass}}{\text{volume}}$)

$$= 1000 \times \pi \left(\frac{y}{2}\right)^2 \Delta y \times 9.8 \times (10 - y + 2)$$

$$= 9800\pi \left(\frac{y}{2}\right)^2 (12 - y)\, \Delta y$$

∴ Work done to empty the half-full tank

$$= \int_0^5 9800\pi \left(\frac{y}{2}\right)^2 (12-y)\, dy$$

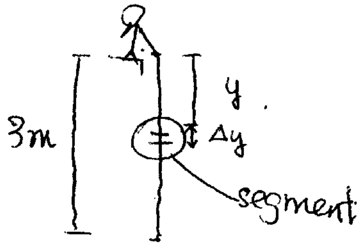
$$= 2450\pi \int_0^5 (12y^2 - y^3)\, dy$$

$$= 2450\pi \cancel{\int_0^5} \left[4y^3 - \frac{y^4}{4}\right]_0^5$$

$$= 2450\pi \left(4(5)^3 - \frac{5^4}{4}\right)$$

$$= \frac{1684375\pi}{2} = 2.65 \times 10^6 \text{ J}$$

*(27).



mass density $= 4\,kg/m$

i.e. 1m of the chain weighs 4 kg

$\therefore$ Δy m of the chain weighs $(4\,\Delta y)$ kg

Work needed to lift the segment of the chain

$=$ weight of the chain $\times$ distance needed to lift

$=$ mg $\times$ distance

$= (4\Delta y)(9.81) \times y$

$\therefore$ Work needed to lift the entire chain

$$= \int_0^3 4(9.81)\, y \, dy$$

$$= 4(9.81)\left[y^2\right]_0^3 = 176.4\ J$$