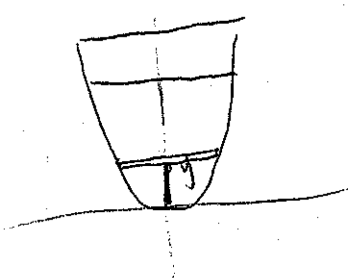
**Group Quiz 2**
**Calc II**
**Summer 2010**

YOUR NAME: ____________________
GROUP MEMBER: ____________________
GROUP MEMBER: ____________________

**Problem 1:** Suppose a dam has the shape of the region bounded by the following:

$$10(1-\cos x),\ -\pi/2 \le x \le \pi/2,\ \text{and } y = 10.$$

Set up an integral to find the pressure on the dam when the water is 8 ft deep.




Constant: $w$

depth of slice $8-y$

width of slice : $2 \cdot \arccos(1-\frac{y}{10})$.

$y = 10(1-\cos x)$

$1 - \frac{y}{10} = \cos x$

$\arccos(1-\frac{y}{10}) = x$

So, I have a slice for $y=0$ up to $y=8$, so

$$\int_0^8 w \arccos(1-\tfrac{y}{10})(8-y)\,dy.$$

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**Problem 2:** The population $P(t)$ of a species grows according logistically and has carrying capacity $A$. What is the population size at which population growth is maximum. Justify your answer using calculus.

I need to take a derivative to find critical points

$$\frac{dP}{dt} \cancel{P(t)} = k\,y\left(1-\frac{y}{A}\right)$$

d product rule.

$$\frac{d}{dt}\frac{dP}{dt} = \frac{d}{dt}\,ky\left(1-\frac{y}{A}\right)$$

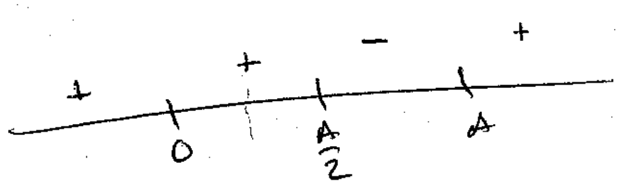
$$= k\frac{dy}{dt}\left(1-\frac{y}{A}\right) + ky\left(\frac{-1}{A}\frac{dy}{dt}\right)$$

$$= \frac{dy}{dt}k\left(1-\frac{2}{A}y\right)$$

$$= \cancel{dy}\; k^2 y\left(1-\frac{y}{A}\right)\left(1-\frac{2}{A}y\right).$$

critical points: $0,\ A,\ \frac{A}{2}$.

To find a max




at $\frac{A}{2}$ y goes from ↗ to ↘

So $\boxed{\frac{A}{2} \text{ is a max}}$

check $y=\frac{A}{4}$ $\quad k^2\,\overset{+}{\frac{A}{4}}\left(\overset{+}{1-\frac{1}{4}}\right)\left(\overset{+}{1-\frac{1}{2}}\right)$

$y=\frac{3A}{4}$ $\quad k^2\,\overset{+}{\frac{3A}{4}}\left(\overset{+}{1-\frac{3}{4}}\right)\left(\overset{-}{1-\frac{3}{2}}\right)$

$y=2A$ $\quad k^2\,2A\,(1-2)(1-4)$

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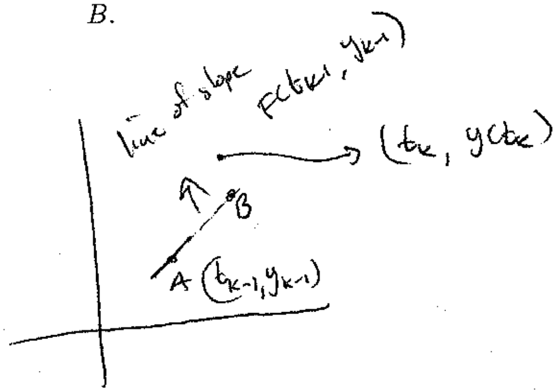
**Problem 3:** Let $y' = F(t, y)$ (with initial condition $y(t_0) = y_0$) and $h$ be the step size in Euler's method.

(a) Recall Euler's method says

$$y_k = y_{k-1} + hF(t_{k-1}, y_{k-1}).$$

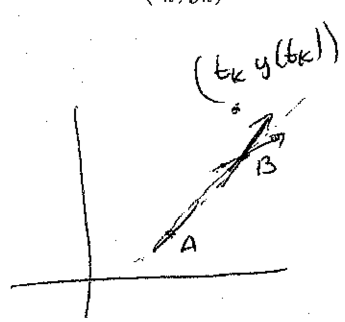
Sketch a graph that includes the point $A = (t_{k-1}, y_{k-1})$, the point $B = (t_k, y_k)$ and an indication of how this method gets you from $A$ to $B$.




(b) Consider the following improvement of Euler's method:

$$y_k = y_{k-1} + \frac{h}{2}(F(t_{k-1}, y_{k-1}) + F(t_k, y_k))$$

Sketch a graph that includes the point $A = (t_{k-1}, y_{k-1})$, the point $B = (t_k, y_k)$ and an indication of how this method gets from $A$ to $B$.




average slope at A and slope at B