# Math 202

5 May 2008
Final Exam (calculator part)

NAME (Print!): ______________________________

Check one: (Dryden): ________
(Pile): ________
(Ryan): ________

| Problem | Points | Score |
|---|---|---|
| 1 | 30 | |
| 2 | 30 | |
| 3 | 20 | |
| 4 | 20 | |
| Total | 100 | |

**Problem 1:** You must show all your work to receive full credit.

(1) Show that the Taylor series for $f(x) = -\ln(1-2x)$ about $a = 0$ is given by

$$-\ln(1-2x) = 2x + 2x^2 + \frac{8}{3}x^3 + 4x^4 + \ldots = \sum_{k=1}^{\infty} \frac{(2x)^k}{k}.$$

(2) Find the radius of convergence for the Taylor series in part (a).

(3) Find the interval of convergence for this Taylor series.

**Problem 2:** A water storage tank has the shape of a cylinder with diameter 10 meters. It is mounted so that the circular cross-sections are vertical. If the depth of the water is 7 meters, what percentage of the total capacity is being used?

**Problem 3:** Suppose you have a well-insulated house that loses heat only through windows; the rate of change of temperature inside the house in degrees Fahrenheit per hour is proportional to the difference in temperature between the outside and the inside. The constant of proportionality is $\frac{1}{29}$. Assume that it is a constant temperature of 10°F outside. On a Thursday at noon the temperature inside the house was 65°F and the heat was turned off until 5 pm.

(1) Write a differential equation which reflects the rate of change of the temperature in the house between noon and 5 pm.
(2) Find the temperature in the house at 5 pm by solving the equation that you found in part (a) analytically.
(3) At 5 pm the heat is turned on. The heater generates an amount of energy that would raise the inside temperature by 2°F per hour if there were no heat loss. Write a differential equation that reflects what happens to the inside temperature after the heat is turned on.
(4) If the heat is left on indefinitely, what temperature will the inside of the house approach?

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**Problem 4:** Answer the following:

(1) Find the area of the region between $y = x^2$ and $y = 2x$.
(2) Find the length of the perimeter of this region.
(3) Find the volume of the solid obtained by rotating this region about the $x$-axis.

**Problem 5:** Determine whether the improper integral converges and, if so, evaluate it.

(1) $\displaystyle\int_0^\infty xe^{-5x}dx$

(2) $\displaystyle\int_1^\infty \frac{dx}{(x+2)(2x+3)}$

**Problem 6:** Sketch the graph of $r = 1 + \cos\theta$. Complete the following table and label the nine points on the curve. Then find the area of the region inside the curve $r = 2$ but outside the curve $r = 1 + \cos\theta$.

| $\theta$ | $0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ | $2\pi/3$ | $3\pi/4$ | $5\pi/6$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $r$ | | | | | | | | | |

**Problem 7:** Compute the following:

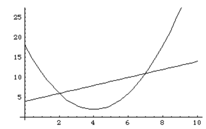
$$\frac{3}{2+i}$$

The answer should be written in the form $a + bi$ for real numbers $a$ ande $b$.

**Problem 8:** Determine whether the following series converges conditionally, convergs absolutely or diverges. Be sure to name whatever test you use and verify that the series satisfies all necessary hypotheses.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$$

**Problem 9:** Suppose the parabola is given by $g(x)$ and the line is $f(x)$. They intersect at $x = 2$ and $x = 7$. Which of the following is the




volume of the solid formed by rotating this region around the $x$-axis

(a) $\int_2^7 2\pi x(f(x) - g(x))\ dx$

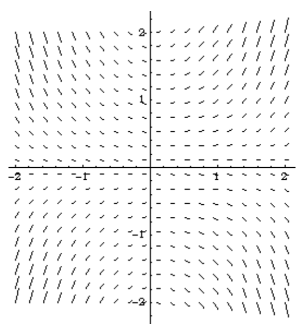
(b) $\int_2^7 f(x) - g(x)\ dx$

(c) $\int_2^7 \pi(f(x) - g(x))^2\ dx$

(d) $\int_2^7 \pi(f(x)^2 - g(x)^2)\ dx$

(e) $\int_2^7 \pi x(f(x) - g(x))\ dx$

**Problem 10:** Match the slope field with the differential equation:




(a) $\frac{dy}{dx} = \frac{x}{y}$

(b) $\frac{dy}{dx} = \frac{-x}{y}$

(c) $\frac{dy}{dx} = \frac{y}{x}$

(d) $\frac{dy}{dx} = \frac{-y}{x}$

(e) $\frac{dy}{dx} = xy$

(f) $\frac{dy}{dx} = -xy$

**Problem 11:** Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\pi^{2n+1}}{4^{2n+1}(2n+1)!}$$

(a) $1/4$

(b) $\sqrt{2}/2$

(c) $-1/4$

(d) $-\sqrt{2}/2$

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**Problem 1:** As usual, you must show your work to receive full credit.

(1) Find the Taylor polynomial of degree three around $x = 0$ for the function
$$f(x) = \sqrt{1 - x}.$$

(2) Use your answer in part (a) to give approximate values to $\sqrt{\frac{1}{2}}$ and $\sqrt{0.9}$.

(3) Which approximation in part (b) is more accurate? Explain why, giving a rigorous mathematical justification (i.e., not just "my calculator tells me that this approximation is more accurate").

**Problem 2:** A cone-shaped water reservoir is 20 m in diameter across the top and 15 m deep. If the reservoir is filled to a depth of 10 m, how much work is required to pump all the water to the top of the reservoir? (The density of water is 1000 g/$m^3$.)

**Problem 3:** In this problem, you will be working with the differential equation

$$y' = y + xy.$$

We are interested in the solution that satisfies the initial condition $y(0) = 1$.

(1) Use Euler's Method with step size 0.1 to estimate $y(0.5)$.
(2) Is your estimate an overestimate or an underestimate? Give justification for your answer using the slope field. You don't need to draw the slope field, but you should graph it on your calculator and describe it as necessary to support your justification.
(3) Find the exact solution to this initial-value problem. Use your solution to find the error in using the approximation to $y(0.5)$ that you used in part (a).

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**Problem 4:** Find the area of the region bounded by the curves $y = \cos x$ and $y = \cos^2 x$ between $x = 0$ and $x = \pi$.

**Problem 5:** Determine whether the improper integral converges and, if so, evaluate it.

(1) $\displaystyle\int_1^{\infty} \frac{\tan^{-1} x}{x^3} dx$

(2) $\displaystyle\int_0^{\pi/2} \sin x \cdot \ln(\cos x)\, dx$

**Problem 6:** Sketch the graph of $r = \cos 3\theta$. Complete the following table and label the nine points on the curve. Then find the area of the region enclosed by the circle $r = 1/2$ and one petal of the curve $r = \cos 3\theta$.

| $\theta$ | $0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ | $2\pi/3$ | $3\pi/4$ | $5\pi/6$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $r$ | | | | | | | | | |

**Problem 7:** Compute the following:

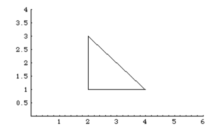
$$(1/2 + \sqrt{3}/2i)^{15}$$

The answer should be written in the form $a + bi$ for real numbers $a$ ande $b$.

**Problem 8:** Determine whether the following converges conditionally, converges absolutely or diverges. Be sure to name whatever test you use and verify that the series satisfies all necessary hypotheses.

$$\sum_{n=1}^{\infty} \frac{2^{3n}}{n^2 5^n}$$

**Problem 9:** Imagine rotating the triangular region around the $x$-axis, the $y$-axis and the line $x = 6$. Which of the following is a list of the



three volumes in increasing order?

(a) $x$-axis, $y$-axis, $x = 6$

(b) $x$-axis, $x = 6$, $y$-axis,

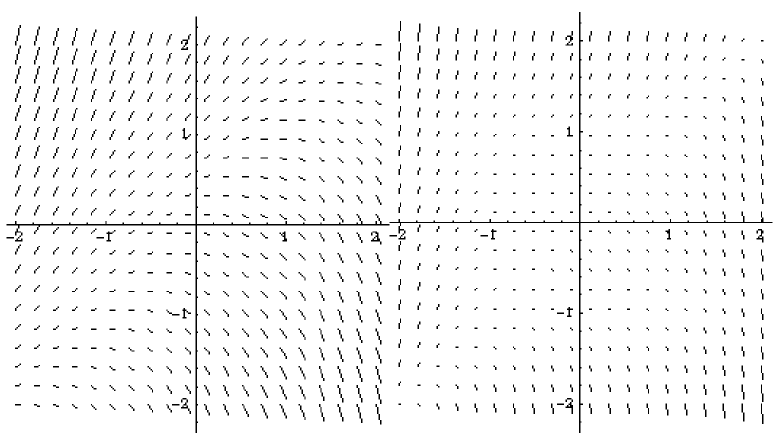
(c) $y$-axis, $x$-axis, $x = 6$

(d) $y$-axis, $x = 6$, $x$-axis

(e) $x = 6$, $x$-axis, $y$-axis

(f) $x = 6$, $y$-axis, $x$-axis

**Problem 10:** One of the two picture below corresponds to the differential equation $y' = y - x$ and the other to $y' = y^3 - x^3$. Identify which is which. Also, on the first one, sketch the solution that satisfies the initial condition $y(0) = 1$.



**Problem 11:** If $\sum_{n=0}^{\infty} c_n(4)^n$ is convergent, then

(a) $\sum_{n=0}^{\infty} c_n(-4)^n$ must converge

(b) $\sum_{n=0}^{\infty} c_n(-4)^n$ may converge

(c) $\sum_{n=0}^{\infty} c_n(-4)^n$ must diverge