Math 202

5 May 2008
Final Practice Exam (calculator part)

NAME (Print!): _____

Check one: (Dryden): _____
(Pile): _____
(Ryan): _____

Problem	Points	Score
1	30	
2	25	
3	25	
4	20	
5	20	
6	25	
7	15	
8	10	
9	10	
10	10	
11	10	
Total	200	

Problem 1: As usual, you must show your work to receive full credit.

(1) (12 points) Find the Taylor polynomial of degree three around $a = 0$ for the function

$$f(x) = \sqrt{1 - x}.$$

(2) (10 points) Use your answer in part (1) to give approximate values to $\sqrt{\frac{1}{2}}$ and $\sqrt{0.9}$.

(3) (8 points) Which approximation in part (2) is more accurate? Explain why, giving a rigorous mathematical justification (i.e., not just "my calculator tells me that this approximation is more accurate").

Problem 2: (25 points) A cone-shaped water reservoir is 20 m in diameter across the top and 15 m deep. If the reservoir is filled to a depth of 10 m, how much work is required to pump all the water to the top of the reservoir? (The density of water is 1000 kg/m^3.)

Problem 3: In this problem, you will be working with the differential equation

$$y' = y + xy.$$

We are interested in the solution that satisfies the initial condition $y(0) = 1$.

(1) (10 points) Use Euler's Method with step size 0.1 to estimate $y(0.5)$.

(2) (5 points) Is your estimate an overestimate or an underestimate? Give justification for your answer using the slope field. You don't need to draw the slope field, but you should graph it on your calculator and describe it as necessary to support your justification.

(3) (10 points) Find the exact solution to this initial-value problem.

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Problem 4: (20 points) Find the area of the region bounded by the curves $y = \cos x$ and $y = \cos^2 x$ between $x = 0$ and $x = \pi$.

Problem 5: (20 points) Determine whether the improper integral given below converges. If it does, evaluate it.

$$\int_1^\infty \frac{\tan^{-1} x}{x^3} dx$$

Problem 6: (25 points) Complete the following table and sketch the graph of $r = \cos 3\theta$. Please label the nine points on the curve. Then write an integral representing the area of the region enclosed by both the circle $r = 1/2$ and one petal of the curve $r = \cos 3\theta$. You do not need to evaluate your integral.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
r									

Problem 7: (15 points) Determine whether the series given below converges conditionally, converges absolutely or diverges. Be sure to name any test(s) you use and verify that the series satisfies all necessary hypotheses.

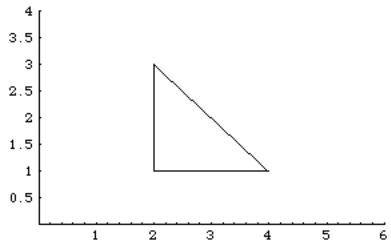
$$\sum_{n=1}^{\infty} \frac{2^{3n}}{n^2 5^n}$$

Problem 8: (10 points) Compute

$$\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{15}$$

and express your answer in the form $a+bi$ for real numbers a and b.

Problem 9: (10 points) Imagine rotating the triangular region shown below around the x-axis, the y-axis and the line $x = 6$. Select the list which gives the three volumes in increasing order.



(a) x-axis, y-axis, $x = 6$

(b) x-axis, $x = 6$, y-axis,

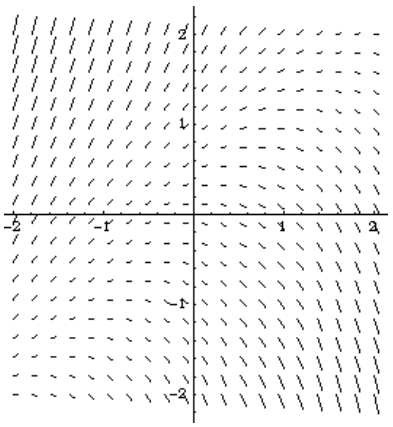
(c) y-axis, x-axis, $x = 6$

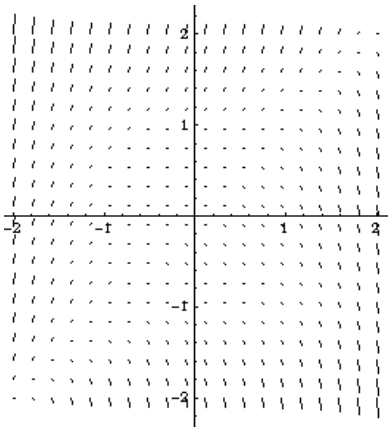
(d) y-axis, $x = 6$, x-axis

(e) $x = 6$, x-axis, y-axis

(f) $x = 6$, y-axis, x-axis

Problem 10: (10 points) One of the two pictures below corresponds to the differential equation $y' = y - x$ and the other to $y' = y^3 - x^3$. Identify which is which. Also, on the first one, sketch the solution that satisfies the initial condition $y(0) = 1$.





Problem 11: (10 points) If $\sum_{n=0}^{\infty} c_n(4)^n$ converges, then

(a) $\sum_{n=0}^{\infty} c_n(-4)^n$ must converge.

(b) $\sum_{n=0}^{\infty} c_n(-4)^n$ may converge.

(c) $\sum_{n=0}^{\infty} c_n(-4)^n$ must diverge.