Math 202

15 April 2008

Third Midterm

NAME (Print!): KEY

Check one: (10am): ______

(11am): ______

Problem	Points	Score
1	20	
2	20	
3	40	
4	20	
Total	100	

Problem 1 (20 points): For each of the following two sums, (i) find the sum exactly and (ii) determine how many terms should be added together to get three decimal places of accuracy.

(a) $\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$

$\frac{A}{n+1} + \frac{B}{n+2} = \frac{1}{(n+1)(n+2)} \Rightarrow A(n+2) + B(n+1) =$

$A + B = 0 \quad 2A + B = 1$

$A = 1, B = -1$

$S_1 = \frac{1}{1} - \frac{1}{2}$

$S_2 = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} =$

$S_N = 1 - \frac{1}{N+1} \qquad \sum_{n=0}^{\infty} \frac{1}{n^2+3n+2} = 1$

S_{N00}

need an N so that

$|1 - S_N| < 0.0005$

$\frac{1}{N+1} < 0.0005 \Rightarrow N+1 > 2000$

$N = 2000.$

(b) $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^n}$

$2 \sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$

$S = \frac{\frac{8}{5}}{1 - \frac{4}{5}} = 8$

~~$S_N = ...^{N+}$~~

$S_N = \frac{\frac{8}{5}\left(1 - \left(\frac{4}{5}\right)^n\right)}{1 - \frac{4}{5}}$

$8 - \frac{\frac{8}{5}\left(1 - \left(\frac{4}{5}\right)^n\right)}{1 - \frac{4}{5}} \Rightarrow$

$8 - 8\left(1 - \left(\frac{4}{5}\right)^n\right) < 0.0005$

$8\left(\frac{4}{5}\right)^n < 0.0005$

$\left(\frac{4}{5}\right)^n < \frac{0.0005}{8}$

$n \ln\frac{4}{5} < \frac{0.0005}{8}$

$n > 43$

$n = 44$

Problem 2 (20 points): In this problem, you will be working with the differential equation

$$y' = y + xy.$$

We are interested in the solution that satisfies the initial condition $y(0) = 1$.

(1) (10 points) Use Euler's Method with step size 0.1 to estimate $y(0.5)$.

(2) (5 points) Is your estimate an overestimate or an underestimate? Give justification for your answer using the slope field. You don't need to draw the slope field, but you should graph it on your calculator and describe it as necessary to support your justification.

(3) (10 points) Find the exact solution to this initial-value problem.

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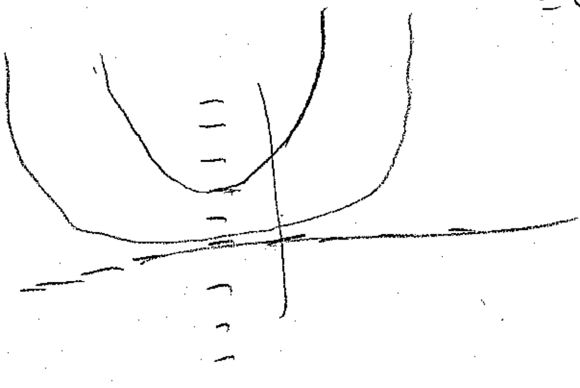
PROBLEM 2.

$y' = y + xy \qquad y' = y(x+1) = F(x,y)$

$= 0$ if $y = 0$ or $x = -1$



$y_k = y_{k-1} + h\,F(x_{k-1}, y_{k-1})$

$y_0 = 1 \quad x_0 = 0$

$y_1 = y_0 + h\,F(x_0, y_0)$
$= 1 + 0.1\,F(0,1)$
$= 1 + 0.1\,(1)$
$= 1.1$

$y_2 = y_1 + 0.1\,F(x_1, y_1)$
$= 1.1 + 0.1\,F(0.1, 1.1)$
$= 1.1 + 0.1\,(1.1^2)$
$= 1.1 + .1(1.21) = 1.221$

$y_3 = 1.221 + 0.1\,F(0.2, 1.221)$
$= 1.221 + 0.1\,(1.465^2)$
$= 1.36752$

$y_4 = 1.36752 + 0.1\,F(0.3)$
$= 1.545$

ⓐ $y_5 = 1.7616$

ⓑ An underestimate because the graph is concave up and the tangent lines are below the ~~actual~~ particular solution

ⓒ $\dfrac{dy}{dx} = y(x+1) \qquad \dfrac{dy}{y} = x+1\,dx$

$\ln y = x^2 + x + C$

$y = Ce^{\cancel{x}^2 + x}$

$y = e^{x^2 + x}$

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Problem 3 (40 points): Determine whether the following series **CONVERGE** or **DIVERGE**. If the series has negative terms, be sure to distinguish between **ABSOLUTE** and **CONDITIONAL** convergence. Be sure to **IDENTIFY** the test you use to justify each assertion that you make and be sure that the **HYPOTHESES** of the test are met.

(a) $\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2+n}$

Since $1 < \cos(n/2) < -1$, $0 \leq |\cos(n/2)| < 1$.

So

$$0 \leq \sum_{n=1}^{\infty} \frac{|\cos(n/2)|}{n^2+n} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

The RHS converges by the p-test $(2>1)$

so the series $\sum \frac{\cos(n/2)}{n^2+n}$ converges absolutely by the comparison test

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$

$$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{\sqrt{n(n+1)}}} = \lim_{n\to\infty} \frac{\sqrt{n^2+n}}{n}$$

$$= \lim_{n\to\infty} \frac{n\sqrt{1+\frac{1}{n^2}}}{n}$$

$$= \lim_{n\to\infty} \sqrt{1+\frac{1}{n^2}} = 1 > 0.$$

By the limit comparison test,

$\sum \frac{1}{\sqrt{n(n+1)}}$ converges

$\sum \frac{\sqrt{n}}{n+5}$

$\int_1^\infty \frac{\sqrt{x}}{x+5}\,dx = \int_1^\infty \frac{u\,(2u\,du)}{u^2+5}$

$u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

$= 2\int_1^\infty \frac{u^2}{u^2+5}\,du$

$= 2\int_1^\infty 1 - \frac{5}{u^2+5}$ diverges.

diverges by the integral test. $\rightarrow$

(c) $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{n+5}$

$\frac{\sqrt{n}}{n+5}$ is decreasing eventually $\left(\left(\frac{\sqrt{x}}{x+5}\right)' = \frac{\frac{(x+5)}{2\sqrt{x}} - \sqrt{x}}{(x+5)^2} = \frac{-x+5}{\sqrt{x}\,(x+5)^2} \right)$

is positive

conditionally converges

$\lim_{n\to\infty} \frac{\sqrt{n}}{n+5} = \lim_{n\to\infty} \frac{n\sqrt{\frac{1}{n^2}}}{n\left(1+\frac{5}{n}\right)} = 0$

by the alternating series test it converges

(d) $\sum_{n=1}^{\infty} \frac{2n}{8n-5}$

$\lim_{n\to\infty} \frac{2n}{8n-5} \neq 0$ so the ~~limit~~ series diverges by the divergence test.

(e) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$

$\lim_{n\to\infty} \frac{\frac{\ln(n)}{n^2}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{\ln(n)}{\sqrt{n}}$

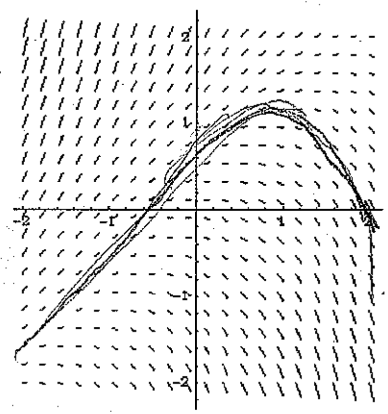
$\underset{LH}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2\sqrt{n}}}$

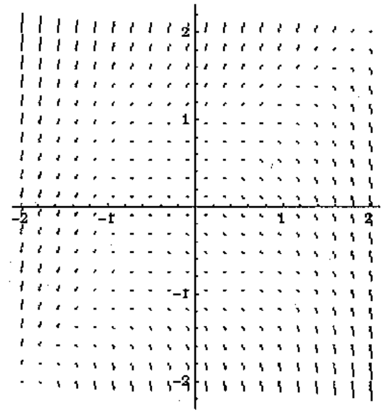
$= \lim_{n\to\infty} \frac{2\sqrt{n}}{n}$

$= \lim_{n\to\infty} \frac{2n\sqrt{\frac{1}{n^2}}}{n}$

$= 0$

Problem 4 (20 points): One of the two pictures below corresponds to the differential equation $y' = y - x$ and the other to $y' = y^3 - x^3$. Identify which is which. Also, on the first one, sketch the solution that satisfies the initial condition $y(0) = 1$.





$y' = y^3 - x^3$

because at $(2, 0)$ very steep, negative

$y' = y - x$

at $(2, 0)$ less