Math 202

12 February 2008

First Midterm

NAME (Print!): ________________________________

Check one: (1pm): ______

(2pm): ______

Problem	Points	Score
1	30	
2	20	
3	20	
4	20	
5	10	
Total	100	

Problem 1 (30 points): Suppose rain falls for ten hours at a velocity

$$v(t) = t^2(10-t)^2 \; cm/hr.$$

(a) In units cm^3/hr what is the flow rate of water into a pool that's $1\ m^2$? Use this to **set up but not solve** an integral that computes the total volume of water added to the pool after the 10 hours of rain.

(b) Suppose that the rain is acid rain and that the concentration of some pollutant in the rain is $2\ ng/cm^3$ at the beginning and is $1\ ng/cm^3$ at the end of the rain. **Set up but do not solve** an integral that computes the total amount of pollutants in the pool after the ten hours of rain.

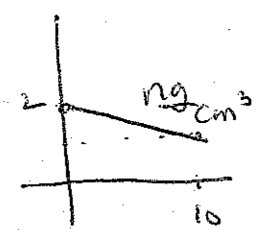
(c) Justify parts (a) and (b) by relating your integral to a Riemann sum.

a) I want units $\frac{cm^3}{hr}$ so

flow rate $= t^2(10-t)^2 \frac{cm}{hr} \; (100)^2 \; cm^2$

$$\boxed{\int_0^{10} (100)^2 \, t^2 (10-t)^2 \, dt}$$

b)



Find $\int_0^{10} \frac{ng}{hr} \, dt$ to get total mass.

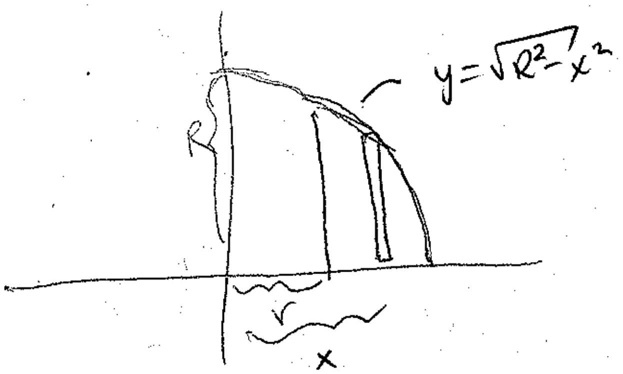
$$\boxed{\int_0^{10} \underbrace{\left(2-\frac{t}{10}\right)}_{\frac{ng}{cm^3}} \underbrace{t^2(10-t)^2 \, 100^2}_{\frac{cm^3}{hr}} \, dt}$$

c) Break 0–10 hr time range into little bits.
volume increase in dt hours is $t^2(10-t)^2 \, 100^2 \, dt$
Integrate from 0 to 10.

Problem 3 (20 points): Suppose you have a sphere of radius R and you drill out a hole through the center of radius $r < R$. **Set up** two integrals that represent the volume of the remaining solid and **solve** one of them.

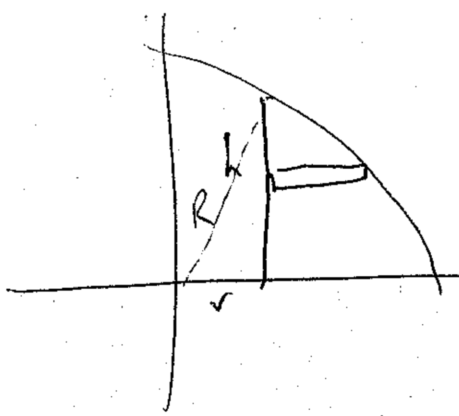
By shells:

$$2\left(2\pi\int_r^R x\sqrt{R^2-x^2}\quad dx\right)$$



I need to find h:

$R^2=r^2+h^2$ so $h=\sqrt{R^2-r^2}$



$$2\left(\int_0^{\sqrt{R^2-r^2}} \pi\sqrt{R^2-y^2} - \pi r^2 \quad dy\right)$$

Problem 2 (20 points): Find the following integrals

(a) $\int_0^{\pi/2} \sin x \cos x \; dx$ $= \int_0^1 u \, du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2}$.

$u = \sin x$ ~~dx~~

$du = \cos x \; dx$

(b) $\int_1^2 \frac{x}{x^2+1} \; dx$ $= \frac{1}{2}\int_1^5 \frac{du}{u} = \frac{1}{2} \ln u \Big|_1^5$

$u = x^2+1$

$du = 2x \, dx$

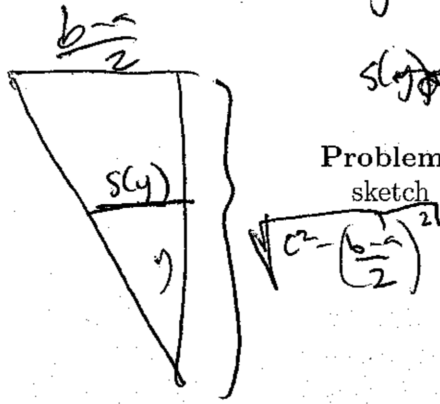
$= \frac{1}{2} \ln 5 - \frac{1}{2} \ln 1$

$= \frac{1}{2} \ln 5$.

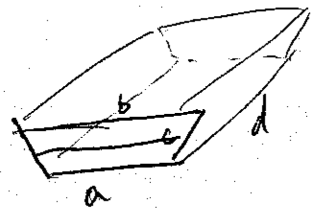
$\ell(y) = a + 2s(y)$

$\frac{y}{s(y)} = \frac{\sqrt{c^2-\left(\frac{b-a}{2}\right)^2}}{\frac{b-a}{2}}$

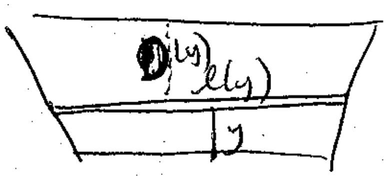
$s(y) = \frac{\frac{b-a}{2}}{\sqrt{c^2-\left(\frac{b-a}{2}\right)^2}}\, y$



Problem 4 (20 points): Suppose you have a tank such as in the sketch below. The trapezoidal faces are perpendicular to the ground



and the bottom side is of length a, the top side is of length b and the remaining sides are of length c. The tank has length d. Find the work required to empty the tank of water by removing the water from the top.



Slice

$W = Fd$

$= \text{mass} \cdot \text{gravity} \cdot \text{distance}$

$= \text{density} \cdot \text{volume} \cdot \text{gravity} \cdot D(y)$

$= 9800\, \ell(y)\, d\, dy\, D(y)$

$= 9800\, \ell(y)\, d\, D(y)\, dy.$

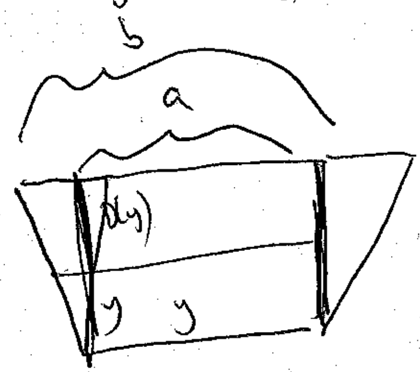
$= 9800\left(\sqrt{c^2-\left(\frac{b-a}{2}\right)^2} - y\right) d\, \ell(y)\, dy.$

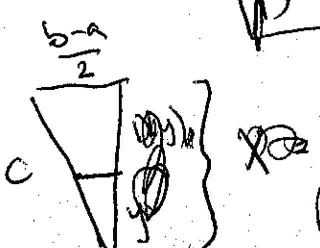
$= 9800\left(\sqrt{c^2-\left(\frac{b-a}{2}\right)^2} - y\right) d \left(a + 2\frac{\frac{b-a}{2}}{\sqrt{c^2-\left(\frac{b-a}{2}\right)^2}}\, y\right)$

$\int_0^{\sqrt{c^2-\left(\frac{b-a}{2}\right)^2}} \dots$

Need to find $\ell(y)$ and $D(y)$

$D(y) =$





$(y+D(y))^2 + \left(\frac{b-a}{2}\right)^2 = c^2 \Rightarrow y^2 + \cancel{2D(y)y} + D(y)$

$D(y) = \sqrt{c^2-\left(\frac{b-a}{2}\right)^2} - y$

Problem 5 (10 points): The total amount of radioactive material present in the atmosphere at time T is

$$A(T) = \int_0^T Pe^{-rt}\, dt$$

According to the UN, currently there are $P = 200$ millirads of material the atmosphere and $r = 0.002$. Estimate the amount of material that will accumulate in the future, assuming these values stay constant.

$$A(T) = \left. \frac{Pe^{-rt}}{-r} \right|_0^T$$

$$\boxed{= \frac{Pe^{-rT}}{-r} + \frac{P}{r}.}$$