Worksheet 1: Algebra Review

1. Below is a list of some "simple" algebra problems. Some of the solutions are correct and some of them are **wrong**! For each problem:
   A. determine if the answer is correct;
   B. determine if there are any mistakes made in solving the problem and list them (**note** that just because the answer is corrrect does not mean there are no mistakes);
   C. if there are mistakes, redo the problem correctly; if there are no mistakes, devise **ANOTHER** correct method to solve the problem.

$$\frac{x^2-1}{x+1} = \frac{x^2+(-1)}{x+1} = \frac{x^2}{x} + \frac{-1}{1} = x-1$$
$$(x+y)^2-(x-y)^2 = x^2+y^2-x^2-y^2 = 0$$
$$\frac{x^{-1}+y^{-1}}{x^{-1}-y^{-1}} = \frac{(x+y)^{-1}}{(x-y)^{-1}} = \frac{x-y}{x+y}$$
$$\frac{x^2y^5}{2x^{-3}} = x^2y^5 \cdot 2x^3 = 2x^5y^5$$

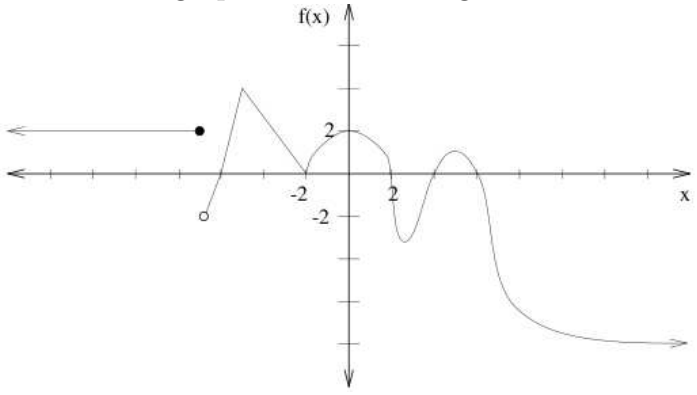
2. Solve the following inequalities. Graph your solution on the real line.

$$1 < |x+1| \leq 2$$
$$0 \leq |x+1| < \infty$$
$$(x-4)(x+1) < 0$$
$$x > \frac{1}{x}$$
$$x^2+x+1 < 0$$

3. Use absolute value to define the following intervals.

$$-7 \leq x \leq 7$$
$$x < -3 \text{ and } x > 3$$
$$-7 < x < 3$$

4. Consider the graph of the following function:



Graph the following:

(1) $f(-x)$
(2) $f(3x)$
(3) $-f(x)$
(4) $3f(x)$
(5) $f(x-1)$
(6) $f(x+1)$
(7) $f(x)+1$

In your own words, describe the process you use to graph

$$af(bx+c)+d$$

where the constants a, b, c, d can be any real number.

5. Graphing (and some calculus) is easier if you can identify symmetries in functions.
   (1) A function is symmetric with respect to the y-axis if $f(x) = f(-x)$. Draw the graph of such a function. A function is symmetric about the origin if $f(-x) = -f(x)$. Draw the graph of such a function. Also, write down examples of such functions.
   (2) Write down a condition for $f(x)$ to be symmetric around the line $x = a$. Draw the graph of such a function. Write down a condition for $f(x)$ to be symmetric around the point $(a, 0)$ on the x-axis. Draw the graph of such a function. Also, write down examples of such functions.