

Math 201  
September 21, 2010  
First (Sample) Midterm

NAME (Print!): \_\_\_\_\_

Check one: (10am): \_\_\_\_\_  
(11am): \_\_\_\_\_

Problem	Points	Score
1	30	
2	30	
3	20	
4	20	
Total	100	

**Problem 1 (30 points):** The signum function, denoted by  $\operatorname{sgn}$  is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

For this function,

- (1) Sketch its graph.
- (2) Find each of the following limits or conclude that it doesn't exist.

Justify your conclusions in words.

- (a)  $\lim_{x \rightarrow 0^+} \operatorname{sgn} x$
- (b)  $\lim_{x \rightarrow 0^-} \operatorname{sgn} x$
- (c)  $\lim_{x \rightarrow 0} \operatorname{sgn} x$
- (d)  $\lim_{x \rightarrow 0^+} |\operatorname{sgn} x|$

**Problem 2 (30 points):** Answer the following questions about limits.

(1) Use continuity to evaluate

$$\lim_{x \rightarrow 2} \arctan \left( \frac{x^2 - 4}{3x^2 - 6} \right).$$

Justify, when appropriate, why you can use continuity.

(2) Use the Squeeze Theorem to show that

$$\lim_{x \rightarrow 0} x^4 \cos \left( \frac{2}{x} \right) = 0.$$

(3) Find the limit, justifying your steps when appropriate:

$$\lim_{x \rightarrow 8} \frac{\sqrt{x+1} - 3}{x - 8}.$$

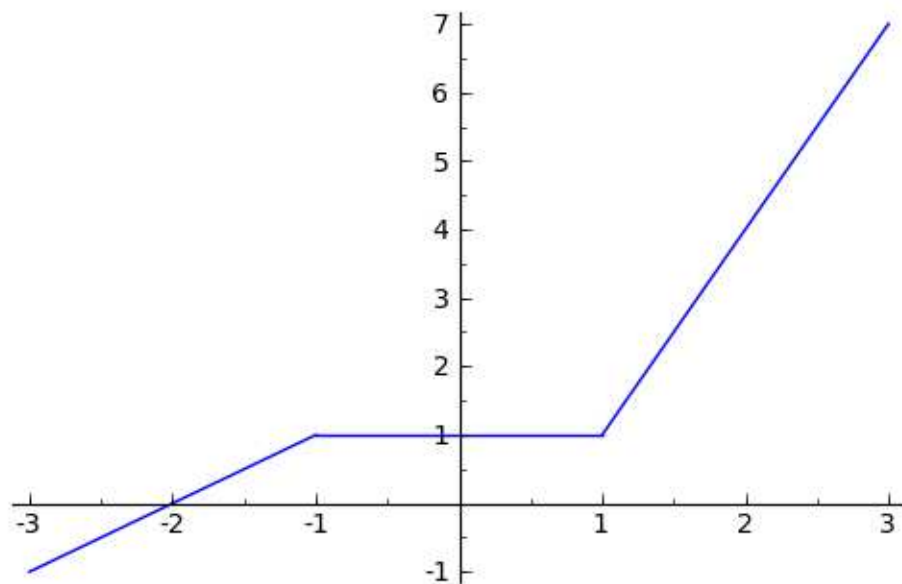
**Problem 3 (20 points):** Answer the following trig questions:

- (1) Express  $\cos(\theta + \frac{\pi}{2})$  and  $\sin(\theta + \frac{\pi}{2})$  in terms of  $\cos \theta$  and  $\sin \theta$ .  
Justify your answer.

- (2) Simplify

$$\tan(\arccos x).$$

**Problem 4 (20 points):** Suppose the graph of  $f(x)$  is given by



(1) Sketch  $f(x + 2)$ .

(2) Sketch  $f(2x + 2)$ .

(THIS PAGE INTENTIONALLY BLANK)