Math 201

16 November 2010

Third (Sample) Midterm

NAME (Print!): KEY.

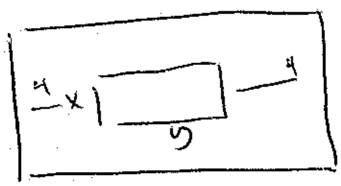
Check one: (12pm): ______

(1pm): ______

Problem	Points	Score
1	20	
2	20	
3	30	
4	20	
5	10	
Total	100	

Problem 1 (20 points): The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of the printed material on the poster is 384 square cm, find the dimensions of the poster with smallest area.

$xy = 384$



$A = (y+8)(x+12)$

I need A as a function of x, not x & y.

So $y = \frac{384}{x}$ and

$A(x) = \left(\frac{384}{x} + 8\right)(x+12) = 384 + \frac{4608}{x} + 8x + 96$

$= 482 + 8x + \frac{4608}{x}$

$A'(x) = 8 - \frac{4608}{x^2}$ CV: $x=0$ $x = \pm 24$.

So Throw away $x=0$, -24

~~Domain for A(x).~~ Open interval. $(0,\infty)$

	$(0, 24)$	$(24, \infty)$
A'	$-$	$+$
A	$\searrow$	$\nearrow$

So a min at $x=24$, $y=16$

So a poster of size $36, 24$ is the smallest.

Problem 2 (20 points): Verify the linear approximation

$$\sqrt[3]{1-x} \approx 1 - x/3$$

for x near 0. Then determine the values of x for which the linear approximation is accurate to within 0.1.

$$f(x) = (1-x)^{1/3}$$

$$f'(x) = -\frac{1}{3}(1-x)^{-2/3}$$

linear approximation is $\quad y - f(0) = f'(0)(x-0)$

$$y - 1 = -\frac{1}{3}(x-0)$$

$$y = 1 - \frac{x}{3}.$$

I need

$$\left|(1-x)^{1/3} - \left(1 - \frac{x}{3}\right)\right| < .1$$

so on my calculator I graph

$$\left|(1-x)^{1/3} - \left(1 - \frac{x}{3}\right)\right|$$

and trace to find the ~~find~~ the x that give be the inequality.

and find $-1.2 < x \le .73$

Problem 3 (30 points): Without using your calculator, graph the function $f(x) = \frac{x^2}{\sqrt{x+1}}$. Be sure to indicate clearly the domain of the function, vertical and horizontal asymptotes (or tell me why there aren't any), local minima and maxima (or tell me why there aren't any), inflection points (or tell me why there aren't any). ~~Take derivatives to show me that you know how to, but~~ Use the following simplified forms for the derivatives:

$$f'(x) = \frac{x(3x+4)}{2(x+1)^{3/2}} \text{ and } f''(x) = \frac{3x^2+8x+8}{4(x+1)^{5/2}}.$$

domain

$x > -1$

as $x \to \infty$ $f(x) \to \infty$

because $x^2 \to \infty$ faster than $\sqrt{x+1}$

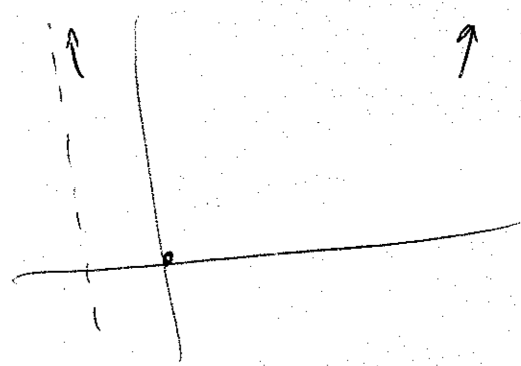
as $x \to -1^+$

$f(x) \to +\infty$

CV for $f'(x)$: $0, -\frac{4}{3}, -1$

	$(-1,0)$	$(0,\infty)$
$f'(x)$	$-$	$+$
$f(x)$	↘	↗

So a min at $x=0$



Possible inflection pts at

$x=-1$

or when

$3x^2+8x+8=0$

But $\frac{-8 \pm \sqrt{64-4\cdot3\cdot8}}{6}$

$\frac{-8 \pm \sqrt{-32}}{6}$ so $3x^2+8x+8$ is never 0

So no inflection points.

Problem 4 (20 points): Without using your calculator, find the following limits. Use l'Hospital's Rule where appropriate. If the Rule doesn't apply, explain why. If there's an easier way to find the limit, feel free to use it. Justify the steps you make.

- $$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$$

$\frac{0}{0}$ so indeterminate

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} \underset{LH}{=} \lim_{x \to 0} \frac{e^x - 1}{2x}$$

$\frac{0}{0}$ indeterminate

$$\underset{LH}{=} \lim_{x \to 0} \frac{e^x}{2}$$

$$= \frac{1}{2}$$

- $$\lim_{x \to 1^+} \frac{x}{x-1} + \frac{1}{\ln x}$$

$$\lim_{x \to 1^+} \frac{(\ln x)\, x + x - 1}{(x-1)\ln x} = \lim_{x \to 1^+} \frac{x \ln x + x - 1}{x \ln x - \ln x}$$

$\frac{0}{0}$ so indeterminate form

$$\underset{LH}{=} \lim_{x \to 1^+} \frac{\frac{1}{x} x + \ln x + 1}{x \frac{1}{x} + \ln x - \frac{1}{x}}$$

$$= \lim_{x \to 1^+} \frac{2 + \ln x}{1 + \ln x - \frac{1}{x}} = +\infty.$$

$= \infty$

Problem 5 (10 points): Let $f(x) = \frac{x+1}{x-1}$. Prove that there is no value of c so that $f(2) - f(0) = f'(c)(2-0)$. Why does this not contradict the Mean Value Theorem? Be precise and justify your argument.

$$\frac{f(2)-f(0)}{2-0} = \frac{3 - -1}{2} = 2$$

$f'(c) = 2$?

$$f'(x) = (x-1)^{-1} - (x+1)(x-1)^{2}$$

$$= \frac{(x-1) - (x+1)}{(x-1)^2}$$

$$= \frac{-2}{(x-1)^2}$$

So $f'(x) < 0$ so there is no c s.t. $f'(c) = 2$.

Doesn't violate the MVT since $f(x)$ is not continuous on $(0,2)$.