

Math 201  
October 26, 2010  
Second (Sample) Midterm

NAME (Print!): \_\_\_\_\_

Check one: (12pm): \_\_\_\_\_  
(1pm): \_\_\_\_\_

Problem	Points	Score
1	20	
2	20	
3	30	
4	20	
5	10	
Total	100	

**Problem 1 (20 points):** Newton's Law of Gravitation states that the magnitude  $F$  of the force exerted by a body of mass  $m$  on a body of mass  $M$  is

$$F = \frac{GMm}{r^2}$$

where  $G$  is the gravitational constant and  $r$  is the distance between the two bodies.

- (a) Find  $\frac{dF}{dr}$  and explain its meaning. What does the minus sign indicate?
- (b) Suppose that it is known Earth attracts an object with a force that decreases at the rate of 2 N/km when  $r = 20,000$  km. How fast does this force change when  $r = 10,000$  km.

**Problem 2 (20 points):** At noon ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and Ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm.

**Problem 3 (30 points):** Find  $\frac{dy}{dx}$  for each of the following:

(a)  $\tan(x - y) = \frac{y}{1+x^2}$

(b)  $y = 2^{3x^2}$

(c)  $y = x^{e^x}$

**Problem 4 (20 points):** Prove the following differentiation rules:

(a) Using the limit definition of the derivative, prove  $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ .

(b) Show that for any real number  $n$  we have  $\frac{d}{dx}x^n = nx^{n-1}$ .

**Problem 5 (10 points):** Find an equation of the tangent line to the curve  $y = 4\sin^2(x)$  at the point  $(\pi/6, 1)$ .

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