Math 201

September 21, 2010
First (Sample) Midterm

NAME (Print!): KEY

Check one: (10am): ______
(11am): ______

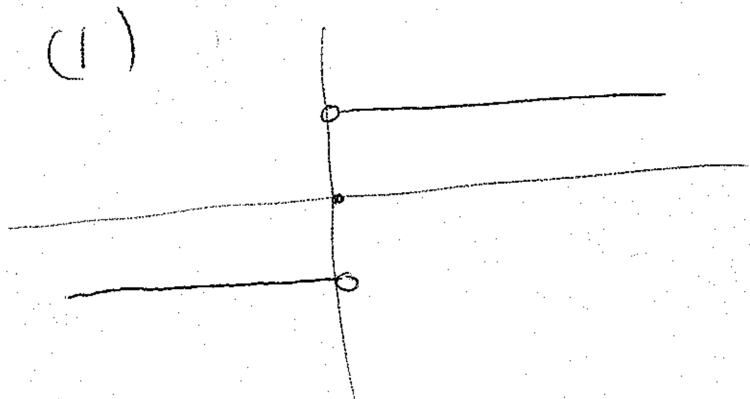
Problem	Points	Score
1	30	
2	30	
3	20	
4	20	
Total	100	

Problem 1 (30 points): The signum function, denoted by sgn is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

For this function,
(1) Sketch its graph. ~~and explain why justify.~~
(2) Find each of the following limits or explain why it doesn't exist. Justify your answer in words.
   (a) $\lim_{x\to 0^+} \operatorname{sgn} x$
   (b) $\lim_{x\to 0^-} \operatorname{sgn} x$
   (c) $\lim_{x\to 0} \operatorname{sgn} x$
   (d) $\lim_{x\to 0^+} |\operatorname{sgn} x|$

(1)



2a 1

2b -1

2c does not exist because $\lim_{x\to 0^+} \operatorname{sgn} x \neq \lim_{x\to 0^-} \operatorname{sgn} x$

2d 1

Problem 2 (30 points): Answer the following questions about limits.

(1) Use continuity to evaluate

$$\lim_{x\to 2} \arctan\left(\frac{x^2-4}{3x^2-6}\right).$$

Justify, when appropriate, why you can use continuity.

$\frac{x^2-4}{3x^2-6}$ is continuous everywhere except at $x = \pm\sqrt{2}$.

arctan is continuous at 2 because tan is continuous at 2.

So the composition of the two is continuous and

$\lim_{x\to 2} \arctan\left(\frac{x^2-4}{3x^2-6}\right) = \arctan(0) = 0.$

(2) Use the Squeeze Theorem to show that

$$\lim_{x\to 0} x^4 \cos\left(\frac{2}{x}\right) = 0.$$

$u(x) = x^4$

$l(x) = -x^4$.

$-1 \leq \cos\frac{2}{x} \leq 1$

① $-x^4 \leq x^4 \cos\frac{2}{x} \leq x^4$

② $\lim_{x\to 0} x^4 = 0$ by substitution

$\lim_{x\to 0} -x^4 = 0$ by substitution.

① + ② $\Rightarrow \lim_{x\to 0} x^4 \cos\left(\frac{2}{x}\right) = 0.$

(3) Find the limit, justifying your steps when appropriate:

$$\lim_{x\to 8} \frac{\sqrt{x+1}-3}{x-8}.$$

Can't plug in 8, so

$\lim_{x\to\infty} \frac{\sqrt{x+1}-3}{(x-8)} \frac{(\sqrt{x+1}+3)}{(\sqrt{x+1}+3)}$

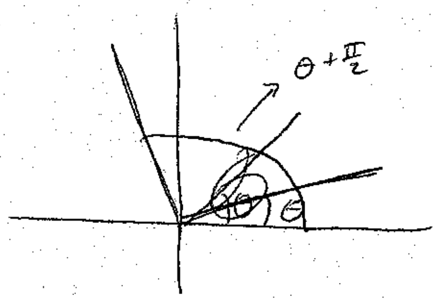
because $x \neq 8$

$= \lim_{x\to 8} \frac{x-8}{(x-8)(\sqrt{x+1}+3)}$

$= \lim_{x\to 8} \frac{1}{\sqrt{x+1}+3} = \frac{1}{6}$ by continuity

Problem 3 (20 points): Answer the following trig questions:

(1) Express $\cos\left(\theta+\frac{\pi}{2}\right)$ and $\sin\left(\theta+\frac{\pi}{2}\right)$ in terms of $\cos\theta$ and $\sin\theta$. Justify your answer.



$\cos\left(\theta+\frac{\pi}{2}\right) = -\sin\theta$

$\sin\left(\theta+\frac{\pi}{2}\right) = \cos\theta$

(2) Simplify

$$\tan(\arccos x).$$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

$\arccos x = \theta, \quad 0 \le \theta \le \pi$

$\tan^2\theta + 1 = \sec^2\theta$

$\tan\theta = \pm\sqrt{\sec^2\theta - 1}$

$= \pm\sqrt{\frac{1}{\cos^2\theta} - 1}$

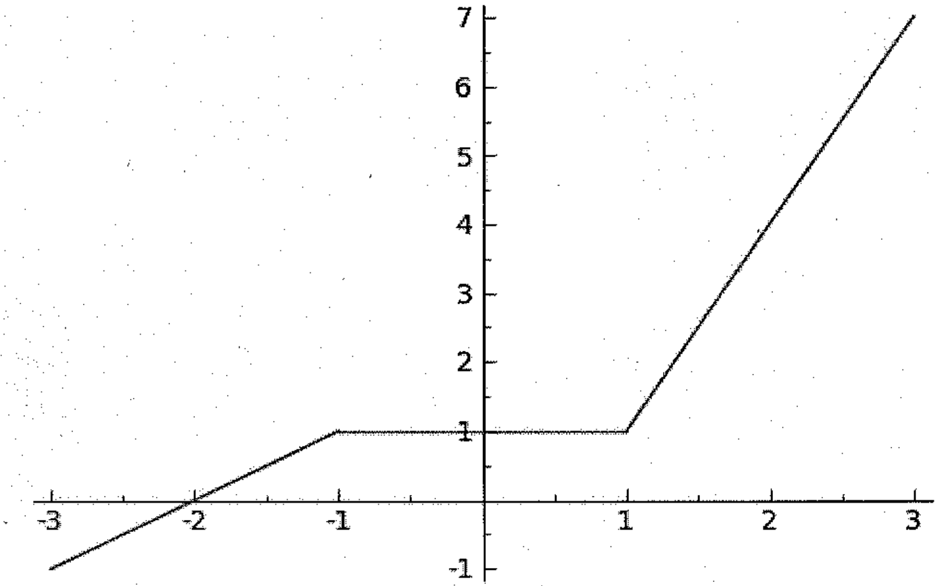
$= \pm\sqrt{\frac{1}{\cos(\arccos \cancel{x})^2} - 1}$

$= \pm\sqrt{\frac{1}{x^2} - 1}$

where $+$ when $0 \le \theta \le \frac{\pi}{2}$

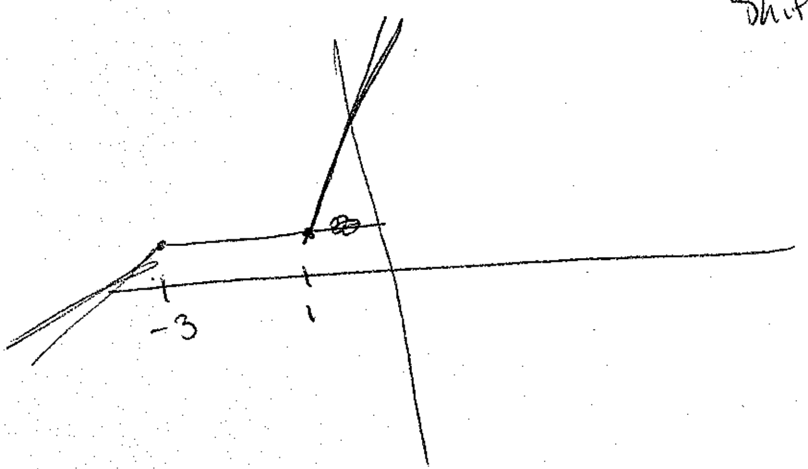
and $-$ when $\frac{\pi}{2} \le \theta \le \pi$

Problem 4 (20 points): Suppose the graph of $f(x)$ is given by

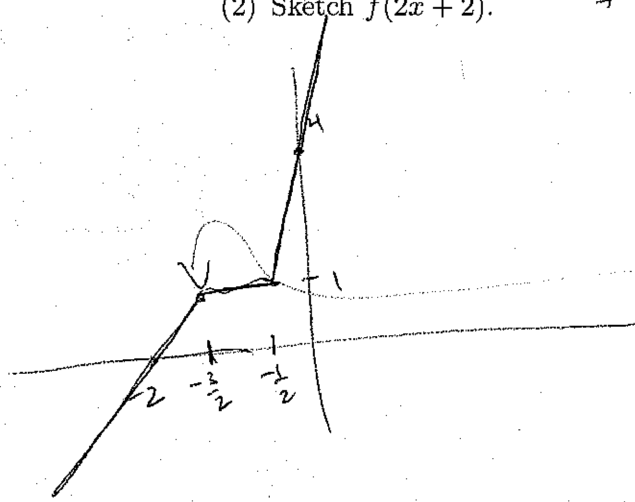


(1) Sketch $f(x+2)$.

Shift two to the left



(2) Sketch $f(2x+2)$. $f(2(x+1))$



I look at the corners of $f(x)$ and figure out what x I need to plug into $f(2x+2)$ to get $f(x)$ at the corner.

e.g. $f(-1)=1$
so find an x so that
$2x+2=-1 \Rightarrow x=-\frac{3}{2}$

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