Proving the power rule (part I)

The book states the following theorem

Theorem 1. *For all exponents n, if $f(x) = x^n$, then $f'(x) = nx^{n-1}$.*

It then provides the following proof (here you fill in the reasons).

Lemma 2. *For an integer n greater than 1,*

$$x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \cdots + a^{n-2}x + a^{n-1}).$$

Proof. Prove this by multiplying out the right hand side and seeing that it's equal to the left hand side.

□

Proof of Theorem 1. Use Lemma 2 and the definition of the derivative in the form

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

to prove Theorem 1.

□

1: Does the proof you just found prove Theorem 1? Why or why not?

In the coming weeks we'll formulate a complete proof of Theorem 1. We will use the **quotient** rule to prove Theorem 1 when the exponent is a negative integer; we will use the **chain** rule to prove Theorem 1 when the exponent is a rational number; we will use the **chain** rule again to prove Theorem 1 when the exponent is an arbitrary real number.

Theorem 3. *If $f(x) = b^x$, then $f'(x)$ is proportional to b^x.*

Proof of Theorem 3. By using the definition of the derivative (in terms of h), show that

$$f'(a) = f(a) \left(\lim_{h \to 0} \frac{b^h - 1}{h} \right).$$

□

2: Is what I say in the hint for Theorem 3 enough to prove Theorem 3? Why or why not?

3: Verify numerically that if $f(x) = e^x$, $f'(x) = e^x$. Verify numerically that if $g(x) = 2^x$, $g'(x) = (\ln 2)2^x$.