Math 201

17 December 2008

Final Exam

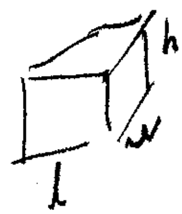
NAME (Print!): ____________________

Check one: (1pm): ______

(2pm): ______

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	30	
6	30	
7	20	
8	20	
9	10	
10	10	
Total	200	

Problem 1 (20 points): According to postal regulations, a carton is classified as "over-sized" if the sum of its height and girth (the perimeter of its base) exceeds 108 in. Find the dimensions of a cartons with a square base that is not over-sized and has maximum volume.



$h + 2l + 2w$

$l = w \leftarrow$ square base

$h + 4l = \cancel{108} = 108$

$V = l^2 h$

$= l^2 (108 - 4l)$

$V = 108 l^2 - 4 l^3$

$V' = 216 l - 12 l^2$

$0 = \cancel{2..}\, l(216 - 12l)$

$= l(216 - 12l)$

$l = 0, \; \frac{216}{12} = \frac{6^3}{12} = \cancel{36}$

$= 3 \cdot 6$

$= 18$

h

$V'(10) > 0$

$V'(100) < 0$ so 18 is a max

$h + 4 \cdot 18 = 108$

$h = 36$

18 in × 18 in × 36 in

gives largest volume

Problem 2 (20 points): Gravel is being dumped onto a conical pile at the rate of 30 cubic ft per minute. The height and radius of the pile are always equal. How fast is the height of the pile increasing when the pile is 10 ft high. The volume of a cone is given by $V = \pi r^2 h/3$. $\leftarrow h = r$

$$\frac{dV}{dt} = 30 \ ft^3/min$$

$$V = \frac{\pi r^2 h}{3} = \frac{\pi h^3}{3}$$

$$\frac{dV}{dt} = 3\pi h^2 \frac{dh}{dt}$$

$$30 = 3\pi (100) \frac{dh}{dt}$$

$$\frac{30}{300\pi} = \frac{dh}{dt}$$

$$\frac{1}{10\pi} = \frac{dh}{dt}$$

$$\boxed{\frac{1}{10\pi} \ ft/min}$$

Problem 3 (20 points): The stopping distance for an automobile (after applying the brakes) is approximately $D(s) = 1.1s + 0.054s^2$ ft, where s is the speed in mph. Find the linear approximation to $D(s)$ and use it to approximate the change in stopping distance per additional mph when $s = 35$.

$D'(s) = 1.1 + 0.108s$

$y - D(35) = D'(35)(x-35)$

Plug in $x = 32$ to find the answer.

Problem 4 (20 points): The traffic flow rate past a certain point on a highway is $q(t) = 3000 + 2000t - 300t^2$ where t is in hours and $t = 0$ is 8 AM. How many cars passed by during the time interval from 8 to 10 AM?

$$\int_0^2 3000 + 2000t - 300t^2 \, dt$$

$$= 3000t + 1000t^2 - 100t^3 \Big|_0^2$$

$$= 6000 + 4000 - 800$$

$$= 9200$$

9200 cars passed in those 2 hours.

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NAME (Print!): ______________________

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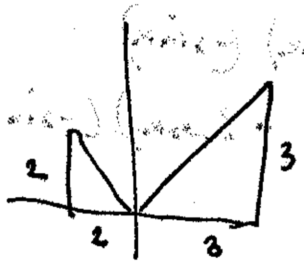
Problem 5 (30 points): Find the following definite integrals:

(a) $\int_{-2}^{3} |x| \, dx$



$$\frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 9 =$$

$$\frac{4}{2} + \frac{9}{2} = \boxed{\frac{13}{2}}$$

(b) $\int_{0}^{\pi/2} \cos^3 x \sin x \, dx$

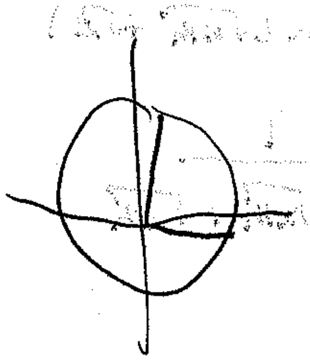
$u = \cos x \qquad du = -\sin x \, dx$

$$\int_0^{\frac{\pi}{2}} \cos^3 x \sin x \, dx = -\int_1^0 u^3 \, du$$

$$= \int_0^1 u^3 \, du = \frac{1}{4}.$$

(c) $\int_{-1/2}^{\sqrt{3}/2} \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x \Big|_{-\frac{1}{2}}^{\sqrt{3}/2}$

$$= \arcsin \frac{\sqrt{3}}{2} - \arcsin -\frac{1}{2}$$



$$= \frac{\pi}{3} - \frac{-\pi}{6} = \frac{\pi}{2}.$$

Problem 6 (30 points): Find derivatives of the following functions:

(a) $f(x) = \int_{-6}^{\cos x} (t^4 - 3t)\ dt$

$f(x) = A(\cos x)$ where $A(x) = \int_{-6}^{x} t^4 - 3t\,dt$

$$f'(x) = A'(\cos x)(-\sin x)$$
$$= (\cos^4(x) - 3\cos x)(-\sin x)$$

(b) f(x)=$\tan^3 x + \tan(x^3)$

$$f'(x) = 3\tan^2(x)\sec^2(x) + (\sec^2 x)(3x^2)$$

(c) $f(x) = \sqrt{x}$ – find this one by using the definition of derivative and without L'Hopital's Rule

$$\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x+h}}$$

$$= \frac{1}{2\sqrt{x}}$$

Problem 7 (20 points): Find the following limits

(a) $\lim_{x\to-\infty} x \sin \frac{1}{x}$

$-\infty \cdot 0$ indeterminate; so LH

$$\lim_{x\to-\infty} \frac{\sin(\frac{1}{x})}{\frac{1}{x}} = \lim_{x\to-\infty} \frac{\cos(\frac{1}{x})(\frac{-1}{x^2})}{\frac{-1}{x^2}}$$

$$= \lim_{x\to-\infty} \cos(\tfrac{1}{x})$$

$$= 1$$

(2)

(b) $\lim_{x\to 0} x \sin \frac{1}{x}$

$$-1 \le \sin\frac{1}{x} \le 1$$

(4)

$$-x \le x \sin\frac{1}{x} \le x$$

$$\lim_{x\to 0} -x = 0 = \lim_{x\to 0} x = 0.$$

By the squeeze theorem, then.

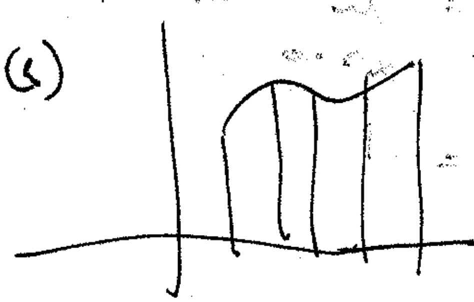
$$\lim_{x\to 0} x \sin\frac{1}{x} = 0.$$

Problem 8 (20 points): State whether the following are true or false, assuming $a < b$. If true, sketch the graph of an example and justify your answer by using the Riemann sum definition of $\int_a^b f(x)\,dx$; if false, sketch the graph of a counterexample.

(a) If $f(x) > 0$, then $\int_a^b f(x)\,dx > 0$

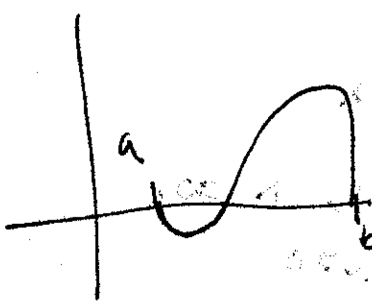
(b) If $\int_a^b f(x)\,dx > 0$, then $f(x) > 0$

(a)



$f(x) > 0$, so each rectangle has positive area and an integral is the sum of positive areas and so is positive

(b)



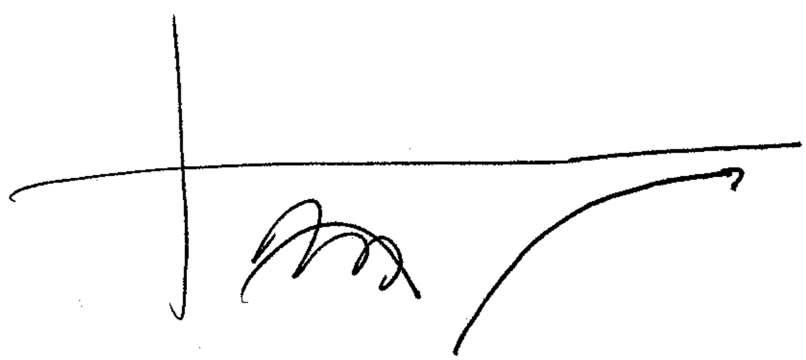
Problem 9 (10 points): Sketch the graph of an increasing function $f(x)$ so that both $f'(x)$ and $A(x) = \int_0^x f(t)\, dt$ are decreasing.

$f'(x) > 0$ increasing

$f''(x) < 0$ concave down

$A'(x) < 0$ below the x-axis

$A'(x) = f(x).$

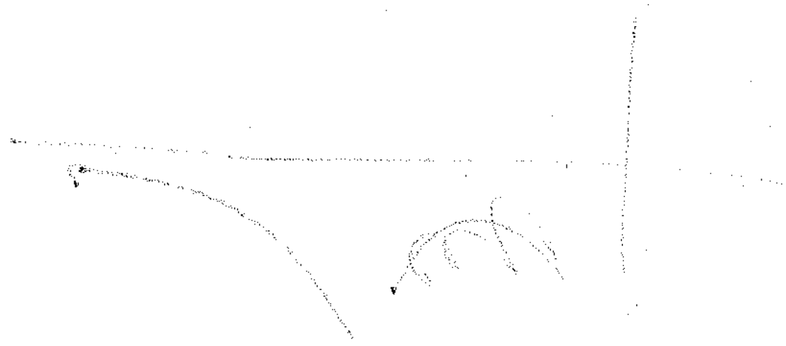


Problem 10 (10 points): Calculate $\int_2^4 f(x)\,dx$ assuming that $\int_0^1 f(x)\,dx = 1$, $\int_0^2 f(x)\,dx = 4$ and $\int_1^4 f(x)\,dx = 7$.

$$\int_2^4 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^4 f(x)\,dx - \int_0^2 f(x)\,dx$$

$$= 1+7-4$$

$$= 4.$$



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