**Group Quiz 3**
**Calc I**
**Fall 2010**

YOUR NAME: ____________________
GROUP MEMBER: ____________________
GROUP MEMBER: ____________________

**Problem 1:** Using the formal definition of the limit, show that

$$\lim_{x\to 2} x+3 = 5.$$

**Problem 2:** Evaluate

$$\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}.$$

Justify your answer using theorems, when appropriate.

**Problem 3:** Justify your answers to the following questions. An intuitive justification with pictures would suffice.

(1) For which values of $x$ is the function

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$

continuous?

(2) For which values of $x$ is the function

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational} \end{cases}$$

continuous?

~~to~~ Let $\varepsilon > 0$. I need a $\delta$ so that if

$$|x-2| < \delta$$

then

$$|f(x) - L| < \varepsilon$$

i.e.

$$|x+3-5| < \varepsilon.$$

~~$\varepsilon$ shoul~~ $\delta$ should be a function of $\varepsilon$.

Now, I see

$$|x+3-5| = |x-2| ,$$ so I can let $\delta = \varepsilon$.

Proof:

Let $\varepsilon > 0$. And suppose $\delta = \varepsilon$.

Now, ~~since~~ assume $|x-2| < \delta$. I want to show $|f(x) - L| < \varepsilon$.

So,

$$|x-2| < \delta \underset{\substack{\text{my choice} \\ \text{of } \delta}}{\Rightarrow} |x-2| < \varepsilon$$

$$\Rightarrow |x+3-5| < \varepsilon$$

$$\Rightarrow |f(x) - 5| < \varepsilon.$$

And I'm done.

$$\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$$

Can't use the Quotient law because denominator is 0 at 2.
Can't use substitution because of the same reason.

So Algebra!

$$\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1} \frac{(\sqrt{3-x}+1)}{(\sqrt{3-x}+1)} = \lim_{x\to 2} \frac{(\sqrt{6-x}-2)(\sqrt{3-x}+1)}{3-x-1}$$

$$= \lim_{x\to 2} \frac{(\sqrt{6-x}-2)(\sqrt{3-x}+1)}{2-x}$$

Can't use QL or substitution so ALGEBRA!

$$= \lim_{x\to 2} \frac{(\sqrt{3-x}+1)(\sqrt{6-x}-2)}{2-x} \frac{(\sqrt{6-x}+2)}{(\sqrt{6-x}+2)}$$

$$= \lim_{x\to 2} \frac{(\sqrt{3-x}+1)(6-x-4)}{(2-x)(\sqrt{6-x}+2)} = \lim_{x\to 2} \frac{(\sqrt{3-x}+1)}{(\sqrt{6-x}+2)} \frac{(2-x)}{(2-x)}$$

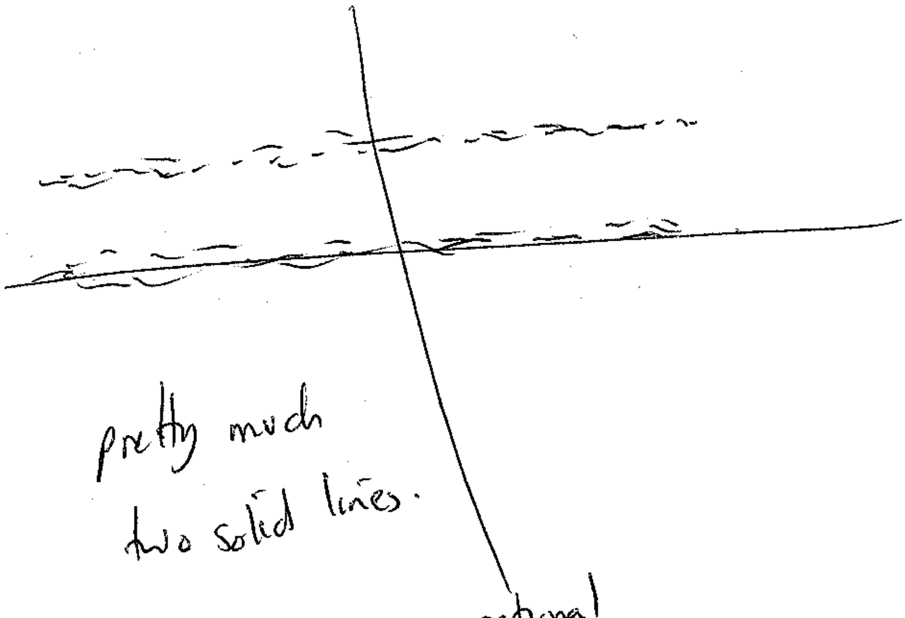
since $x \neq 2$, I can cancel so

$$= \lim_{x\to 2} \frac{\sqrt{3-x}+1}{\sqrt{6-x}+2}$$

function is continuous at 2 so I can substitute

$$= \frac{2}{4} = \frac{1}{2}$$

(1) Let's graph $f(x)$:



| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 1 | 0 |
| 2 | 0 |
| $\frac{1}{4}$ | 0 |
| $\frac{1}{3}$ | 0 |
| $\frac{11}{200}$ | 0 |
| .123 | 0 |
| .123456... | 1 |

pretty much two solid lines.

So as $x \to$ rational

$f(x) \to 0$
and
$f(x) \to 1$

so the limit doesn't exist. In particular the limit is ~~it's~~ not zero, as it should be if ~~f(x)~~ $f(x)$ were continuous there.

Also as $x \to$ irrational

$f(x) \to 0$ and $f(x) \to 1$ while it should go $\to 1$. not continuous there.

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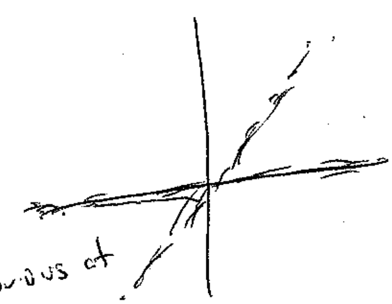
(2) Same kind of argument as above



$\lim_{x\to 0} f(x) = 0$ and $f(0) = 0$, so it's continuous at 0.

At other $x$'s, $x \neq 0$, the graph is trying hard to be and 0. So the limit doesn't exist there. Could also have used the Squeeze Theorem at 0.