Group Quiz 2
Calc I
Fall 2010

YOUR NAME: _____
GROUP MEMBER: _____
GROUP MEMBER: _____

Problem 1: Answer these by hand (but feel free to check your answer with a calculator).

(1) $\sin^{-1}(-\sqrt{3}/2)$
(2) $\cos(\sin^{-1}(\frac{5}{13}))$
(3) $\cos(\sin^{-1}(x))$

Justify your answers using definitions, trig identities and/or geometry.

Problem 2: Answer these by hand (but feel free to check your answer with a calculator).

(1) $\log_2 4 \cdot \log_4 2$
(2) $\log_3 2 \cdot \log_2 3$
(3) $e^{2 \ln x}$

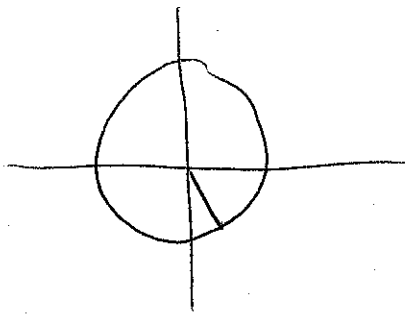
Justify your answers using rules about log and exponentials and definitions.

Problem 3: Sketch the graph of $f(x) = x(1-x)$ over $[0,1]$. Without doing any computations, find

(1) the values of x at which the instantaneuous ROC is positive and those at which it's negative.
(2) the instantaneous rate at $x = \frac{1}{2}$
(3) the average ROC over $[0,1]$

Justify your answers using definitions and the graph.

1.1



since $-\frac{\pi}{2} \leq \sin^{-1}\theta \leq \frac{\pi}{2}$, the unit circle tell us

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{-\pi}{3}.$$

1.2 $\cos\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$. Let $\theta = \sin^{-1}\left(\frac{5}{13}\right)$

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.$$

So $\cos^2\theta + \sin^2\theta = 1$

$$\cos\theta = \pm\sqrt{1 - \sin^2\theta}$$

Since $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ $\cos\theta > 0$, so we choose the $+\sqrt{\ }$.

So $\cos\theta = \sqrt{1 - \sin^2\theta}$

$$= \sqrt{1 - \left(\sin\sin^{-1}\left(\frac{5}{13}\right)\right)^2}$$

$$= \sqrt{1 - \frac{25}{169}}$$

$$= \frac{12}{13}.$$

1.3 Same as 1.2 but with $\frac{5}{13} \longleftrightarrow x$.

2.1

$$\log_2 4 \cdot \log_4 2 = 2 \cdot \frac{1}{2} = 1$$

2.2 $\log_3 2 \ \log_2 3 = 1$

Proof: Let $\log_3 2 = x$

want to show $\log_2 3 = \frac{1}{x}$.

Now $\log_3 2 = x \iff 3^x = 2$

$\iff \log_2 3^x = \log_2 2$

$\iff x \log_2 3 = 1$

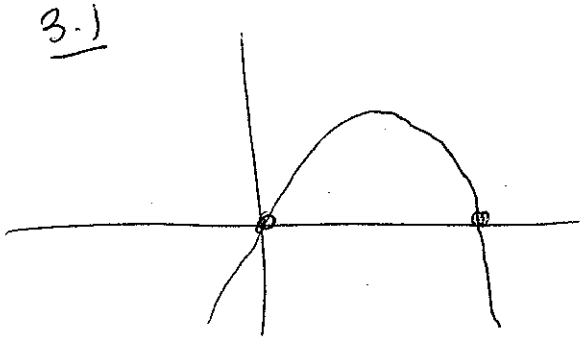
$\iff \log_2 3 = \frac{1}{x}$.

2.3

$$e^{2 \ln x} = e^{\ln(x^2)}$$

$$= x^2.$$

3.1



Tangent lines have positive slope when $x < \frac{1}{2}$, negative slopes when $x > \frac{1}{2}$.

So instantaneous RoC > 0 when $x < \frac{1}{2}$

and instantaneous RoC < 0 when $x > \frac{1}{2}$

3.2 At $x = \frac{1}{2}$, the instantaneous rate of change is 0 because the tangent line at $(\frac{1}{2}, \frac{1}{4})$ has slope 0.

3.3 The average RoC over $[0, 1]$ is 0 because the secant line connecting $(0, f(0))$ and $(1, f(1))$ has slope 0.