

**Problem 1:** Solve the following inequalities. Graph your answer on the number line. Use the geometric argument when possible.

- (a)  $|x + 5| < 2$
- (b)  $|2x + 5| > 2$
- (c)  $|x + 2| \geq x - 7$

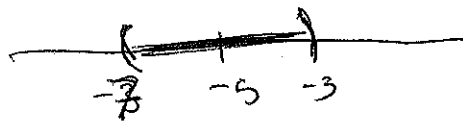
**Problem 2:** Answer the following questions about lines:

- (a) A line of slope  $m = 2$  passes through  $(1, 4)$ . Find  $y$  such that  $(3, y)$  lies on the line.
- (b) Find  $b$  such that  $(2, -1)$ ,  $(3, 2)$  and  $(b, 5)$  lie on a line.
- (c) Find a  $c$  such that  $cx + y = 1$  is vertical.

**Problem 3:** Answer the following questions about quadratics:

- (a) Prove that  $x + \frac{1}{x} \geq 2$  for all  $x > 0$ .
- (b) Find numbers  $x$  and  $y$  with sum 10 and product 24, or show that it's impossible. To do this you could solve a quadratic equation.
- (c) Find numbers  $x$  and  $y$  with sum -1 and product 1, or show that it's impossible. To do this you'll probably have to solve a quadratic.

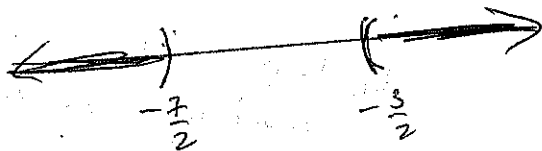
(a) all  $x$  that are no more than 2 units away from -5



(b)  $|2x + 5| > 2 \iff 2|x + \frac{5}{2}| > 2$

$\iff |x + \frac{5}{2}| > 1$

all  $x$  that are at least one unit away from  $-\frac{5}{2}$



(c): If  $x < 7$ , the left hand side is negative, so in that case the inequality is true, since  $|x + 2| \geq 0$ . If  $x \geq 7$ , then  $x + 2 > 0$   
 So  $x + 2 \geq x - 7$ ,  $0 \geq -9$ . Always true. So it works for all  $x$ .

2a

$$y = 2x + b$$

$$4 = 2 \cdot 1 + b$$

$$b = 2$$

~~g~~

$$y = 2 \cdot 3 + 2$$

$$y = 8$$

2b

slope:  $\frac{2 - (-1)}{3 - 2} = 3$

$$y = 3x - 7$$

$$y = 3x + b$$

$$5 = 3 \cdot 2 - 7$$

$$-1 = 3 \cdot 2 + b$$

$$12 = 3b$$

$$b = -7$$

$$b = 4$$

2c

No such  $c$ : Suppose ~~there were a~~ cfo. Then

$$x = \frac{1-y}{c}$$

$$= \frac{1}{c} - \frac{y}{c}$$

I need the term  $\frac{y}{c}$  to disappear and I can't do that. If  $c=0$ , I get  $y=1$  which is horizontal.

3a

Since  $x > 0$ ,

$$x + \frac{1}{x} \geq 2 \iff x^2 + 1 \geq 2x$$

$$\iff x^2 - 2x + 1 \geq 0$$

$$\iff (x-1)^2 \geq 0$$

The last bit is always true. So the inequality is true for all  $x > 0$  disc  $< 0$

3b

$$x + y = 10$$

$$xy = 24$$

$$y = 10 - x$$

$$x(10-x) = 24$$

6, 4

$$10x - x^2 = 24$$

$$x^2 - 10x + 24 = 0$$

$$(x-6)(x-4)$$

3c

$$x + y = -1$$

$$xy = 1$$

$$y = -1 - x$$

$$x(-1-x) = 1$$

$$-x - x^2 = 1$$

$$x^2 + x + 1 = 0$$

$$-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}$$

2

no real solutions

So impossible