Name:

MATH 192 Final Exam Calculator Portion

Show all work to receive credit. Calculators are permitted on this portion of the exam, but please try to make clear what is done by calculator. Note that the last page of this examination is a table of the values of the standard normal variable.

7. A company producing widgets finds that if it produces x widgets it should sell them at the price $\$\ 215x - x^2$ *per widget*. The company has fixed costs of $\$\ 5{,}000$; other than these, if it produces x widgets the cost of production is $\$\ x^2 - 100x$ *per widget*. Determine, by methods of calculus, the number of widgets the company should produce for maximum profit, and justify your answer by showing that it is indeed the maximum.

8. A company finds that the demand function for its product is $q = D(x) = 967 - 25x$, where q is the number of units sold when the price per unit, in cents, is x.

a) Find the elasticity.

b) At what price(s) is the elasticity of demand equal to 1?

c) At what prices is the elasticity of demand elastic? Interpret, in practical terms, what this means for a small increase in price.

d) At what price is the revenue a maximum?

9. a) If at the beginning of the year 2000, $ 1,000 was placed in a bank account yielding 5% annual interest compounded continuously, what is the balance at the beginning of 2002?

b) What is the doubling time for the investment of part a)?

c) Consider a different investment: at the beginning of the year 2000, $ 1,000 was placed in an account paying interest compounded continuously, and at the beginning of 2003 the balance in the account was $1,250 . Find a formula for the value of this investment at time t, where t is measured in years since the beginning of 2000.

9. (continued) d) Consider a third investment: what is the future value of a continuous money flow at the rate $R(t) = 300$ (dollars), at annual interest rate 5%, compounded continuously over 4 years?

e) Find the future values of the investments in a), c), and d) at year 4 (indicate clearly which is which, and we assume the third investment began in year 2000). Which is preferable from this point of view?

9. (continued) f) If the annual interest rate is 5% compounded continuously, each of the payouts in year 2004 had a present value in the year 2000. Compute these present values for investments a), c), and d) (indicate clearly which is which). Which investment is preferable from the point of view of present value in the year 2000?

g) One of the present values you computed in f) is, in 20-20 hindsight, completely obvious. Which, and why?

10. The duration of a cash register transaction, in minutes, at a local store is a continuous random variable with a probability density function of the form $f(t) = \frac{3}{760}t^2 - \frac{27}{380}t + \frac{123}{380}$ over the interval $[0, 10]$.

a) What is the probability that a cash register transaction lasts less than 2 minutes?

b) What is the probability that a cash register transaction lasts longer than 8 minutes?

c) What is the expected value of the duration of a cash register transaction?

11. The score s on a certain test in a high school is normally distributed with mean $\mu = 13$ and standard deviation $\sigma = 3$.

a) Find the probability that s falls between 12 and 16; that is, find $P(12 \leq s \leq 16)$.

b) Find the expected value of a score.

c) The school loses some state revenue for each student scoring below 8. For what proportion of the students does the school lose the revenue?