MATH 192 Review Problems for Third Midterm Spring 2009

(1) Soulful Scents has found that the marginal cost of producing x ounces of a new fragrance is given by

$$C'(x) = 0.0005x^2 - 0.1x + 30, \quad \text{for} \quad x \leq 125,$$

where $C'(x)$ is in dollars.

(a) If you want to approximate the total cost of producing 100 ounces of fragrance, disregarding any fixed costs, using five subintervals and the left end point of each subinterval, draw a graph of C' and the corresponding rectangles.

(b) Find an approximation of the total cost using the subintervals and rectangles you just drew.

(c) Suppose now that the fixed costs for producing a new fragrance is \$1,250, find the total cost function.

(2) Raggs, Ltd., determines that its marginal cost, in dollars per dress, is given by

$$C'(x) = -\frac{2}{25}x + 50, \quad \text{for} \quad x \leq 450.$$

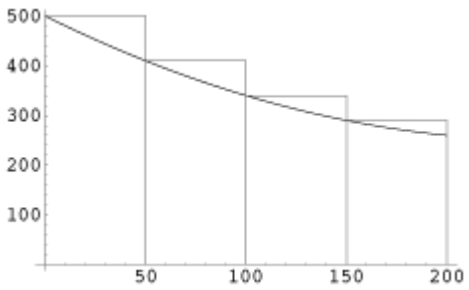
Disregard any fixed costs.

(a) Using a purely geometric argument and a drawing, find the total cost of producing the first 200 dresses.

(b) Using the fundamental theorem of calculus, find the cost of producing 200 dresses.

(3) The marginal cost, in dollars, of producing the xth car stereo is given by

$$C'(x) = 0.004x^2 - 2x + 500.$$



Approximate the total cost of producing 200 car stereos by computing the sum

$$\Sigma_{i=1}^{4} C'(x_i)\Delta x, \quad \text{with} \quad \Delta x = 50.$$

(4) Without a calculator, evaluate the following:

(a)
$$\int 2e^{-5x}\,dx$$

(b)
$$\int 3t^2 - 4t + \frac{7}{t^4}\,dt$$

(c)
$$\int \frac{7e}{x}\,dx$$

(d)

$$\int 2\pi^2 - x^e + e\,dx$$

(e) Find f such that

$$f'(x) = \frac{4}{\sqrt{x}}, \quad f(1) = -5.$$

(5) Compute the following areas without a calculator.
   (a) $\int_2^6 7\sqrt{x} - 2\,dx$
   (b) $\int_{-3}^2 8x^3 + 5e^{-x}\,dx$
   (c) $\int_1^5 \frac{5}{x} + \frac{5}{x^4}\,dx$

(6) The acceleration of gravity on a falling object is -32 ft/sec ^{2}. If I drop a ball from 100 ft off the ground when will it hit the ground.
(Hint: What is the initial velocity? What is the initial position?)

(7) The marginal cost of producing x pounds of cookies is given by $C'(t) = \sqrt{x} + 10x$. What is the total cost of producing 50 pounds of cookies?

(8) Compute the area between the curves.
   (a) $y = x^2 - 2x$ and $y = x$
   (b) $y = x^2$ and $y = \sqrt{x}$

(9) Find the average value of the following functions on the interval indicated.
   (a) $f(x) = 3x^6 + \frac{1}{x}$ on $[-1, 2]$
   (b) $f(x) = 2e^{-2x}$ on $[3, 7]$
   (c) $f(x) = \frac{2}{x^4}$ on $[1, 2]$

(10) Let D be the price in dollars per unit, that consumers are willing to pay for x units of an item, and $S(x)$ is the price, in dollars per unit, that producers are willing to accept for x units. Let

$$D(x) = (x - 10)^2 \qquad S(x) = x^2.$$

   (a) Find the equilibrium point.
   (b) Find the consumer surplus at the equilibrium point.
   (c) Find the producer surplus at the equilibrium point.

(11) Draw a diagram of the typical demand and supply functions (using the convention of section 5.1 that demand and supply are functions of the number of units). On this diagram indicate geometrically the equilibrium point, consumer surplus, producer surplus, and total receipts.
   (a) Explain the meaning of the equilibrium point.
   (b) Explain the meaning of the consumer surplus.
   (c) Explain the meaning of the producer surplus.

(d) Explain the meaning of the total receipts.

(12) Which of these is worth more ten years from now?
   (a) Investing $10,000 today and letting it earn interest for ten years at 7%.
   (b) Investing $1,500 continuously per year for ten years at 7%.

(13) Which of these is worth more today if you can get a 5% return?
   (a) $25,000 in a savings account fifteen years from now.
   (b) Saving $1,100 per year for the next fifteen years.

(14) In 2005 the world production of iron ore was estimated at 1.23 billion metric tons, and production was growing exponentially at the rate of 3% per year. If the production continues to grow at this rate, how much iron ore will be produced from 2005 to 2016?

(15) Evaluate the following integrals

(a)
$$\int_0^\infty e^{-4x}\,dx$$

(b)
$$\int_1^\infty \frac{1}{x^{1/2}}\,dx$$

(16) A company determines that its marginal cost in dollars, for producing x units of a product is given by
$$C'(x) = 2000x^{-3.2}, \quad \text{where} \quad x \geq 1.$$
Suppose that it were possible for the company to make infinitely many units of this product. What would the total cost be?

(17) WeBottleYouDrinkIt sells bottles of water at a stand on a street corner in a medium size town. The company has found that the demand for bottled water is given by
$$q = D(x) = 200 - 100x$$
where q is the number of bottles of water sold at price x. Find each of the following:
   (a) The quantity demanded when the price is 0.75.
   (b) The elasticity as a function of x.
   (c) The elasticity at $x = 0.5$ and $x = 1.50$. Interpret the meaning of these values of the elasticity.
   (d) The value of x at which the elasticity is equal to 1? Interpret the meaning of this price.
   (e) The total revenue function.
   (f) The price x at which total revenue is maximized.