Review Problems for Second Midterm
Math 192
Spring 2009

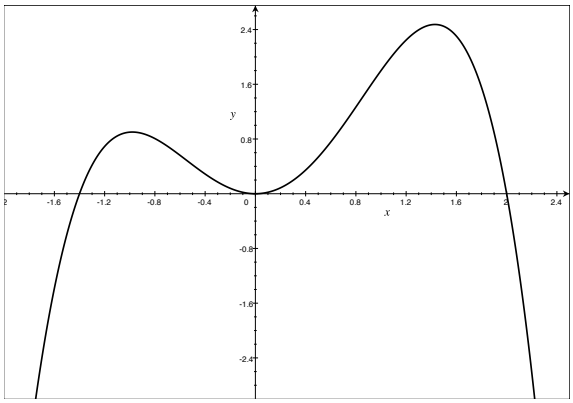
(1) Compute the second derivative of the following functions
   (a) $f(x) = 8x^6 - 13x^4 + 2x$
   (b) $g(x) = \sqrt{5 + x^2}$
   (c) $y = x^{2/3} + \frac{2}{3x+5}$

(2) If the position of an object at time t is given by $s(t) = -10t^2 + 4t + 6$, give the position, velocity, and acceleration at time $t = 2$.

(3) Find and classify the relative and absolute extrema for the following functions on the interval indicated.
   (a) $y = 2x^2 - 4x + 2$, $[0, 3]$ and $(0, \infty)$
   (b) $f(x) = x^3 - 3x + 1$, $[0, 2]$ and $(-\infty, \infty)$
   (c) $y = 2x^4 - 4x^2 + 1$, $[-1, 2]$ and $(-\infty, \infty)$
   (d) $g(x) = x^5 - 3x^2$, $[0, 2]$ and $(-\infty, 2)$
   (e) $y = x - \frac{4x}{x+1}$, $[0, 2]$

(4) The goal of this problem is to sketch the graphs of the functions from the previous problem (except for part (e)). Find the following information about the functions and use it to sketch the graphs.
   (a) critical values
   (b) where the function is increasing and decreasing
   (c) points of inflection
   (d) where the function is concave up and concave down

(5) Use the following graph to determine where the function is increasing, decreasing, and where the critical values are. Also state where the derivative is positive, negative and zero.



(6) A liquid form of antibiotic manufactured by a pharmaceutical firm is sold at \$200 per unit. If the total cost of producing x units is

$$C(x) = 500,000 + 80x + 0.03x^2$$

and if the maximum production capacity is 30,000 units, how many units of this antibiotic must be manufactured and sold to maximize the profit?

(7) The number of bacteria in a culture at time t is given by

$$N(t) = 5000(25 + te^{-t/20}).$$

   (a) Find the largest and smallest number of bacteria in the culture during the time interval $0 \leq t \leq 100$.

(b) At what time during the time interval in the previous question is the number of bacteria decreasing most rapidly?

(8) A commercial cattle ranch currently allows 20 steers per acre of grazing land; on the average steers weigh 2000 lb at market. Estimates by the Agriculture Department indicate that the average market weight per steer will be reduced by 50 lb for each additional steer added per acre of grazing land. How many steers per acre should be allowed in order for the ranch to get the largest possible total market weight for its cattle?

(9) A company selling hockey masks sells 200 per year. It costs \$4 to store a hockey mask for a year. To reorder more masks, there is a fixed cost of \$1 plus \$2.50 for each hockey mask. How many times a year should this store reorder hockey masks and in what lot size, to minimize inventory costs?

(10) A manufacturer of potpourri finds that selling x bags of potpourri generates a revenue of

$$R(x) = 0.006x^3 + 0.02x^2 + 0.4x$$

(a) Find the revenue expected if 25 bags of potpourri are sold.
(b) What is the marginal revenue when 25 bags are sold?
(c) Estimate $R(26)$, $R(28)$ using the previous answer.
(d) Estimate ΔR using differentials if $x = 350$ and $\Delta x = .02$

(11) Find the tangent line to the curve

$$4x^3 - y^4 - 3y + 5x + 1 = 0$$

at $(1, -2)$.

(12) WeBottleYouDrinkIt sells bottles of water at a stand on a street corner in a medium size town. The company has found that the demand for bottled water is given by

$$q = D(x) = 200 - 100x$$

where q is the number of bottles of water sold at price x. Find each of the following:
(a) The quantity demanded when the price is 0.75.
(b) The elasticity as a function of x.
(c) The elasticity at $x = 0.5$ and $x = 1.50$. Interpret the meaning of these values of the elasticity.
(d) The value of x at which the elasticity is equal to 1? Interpret the meaning of this price.
(e) The total revenue function.
(f) The price x at which total revenue is maximized.

(13) Marginal Supply: At a price of x dollars, the supply function for MP3 players is given by

$$q = 75e^{0.004x},$$

where q is in thousands of units.
(a) How many MP3 players will be supplied at a price of \$250? Round to the nearest thousand.
(b) Graph the supply function for $0 \leq x \leq 400$.
(c) Find the marginal supply, $q'(x)$.

(d) Interpret the meaning of the derivative.

(14) Find the equation of the line tangent to the graph of $y = e^{3x}(\ln 4x)$ at $x = 1$.

(15) The percentage P of doctors who prescribe a certain new medicine is

$$P(t) = 100(1 - e^{-0.2t}),$$

where t is the time, in months.

(a) Find $P(1)$ and $P(6)$.

(b) Find $P'(t)$.

(c) How many months will it take for 90% of doctors to prescribe the new medicine?

(16) The cost of a Hershey bar was \$0.05 in 1962 and was increasing at an exponential growth rate of 9.7%.

(a) Predict the cost of a Hershey bar in 2010 and in 2015.

(b) According to the model, in what year would the Hershey bar double in price?

(c) According to the model, during what year would the cost of the bar be \$1?

(17) In 1972, the population of grizzly bears in Yellowstone National Park had shrunk to approximately 190. In 2005, the number of Yellowstone grizzlies had grown to about 610. Find an exponential function that fits the data, and then predict Yellowstone's grizzly bear population in 2012. Round k to three decimal places.

(18) How old is an ivory tusk that has lost 40% of its carbon-14?

(19) Using the tools for graphing that you have learned in calculus, graph

$$f(x) = x \ln x.$$

(20) A certain radioactive element has a decay rate of 7% per day, that is, $\frac{dA}{dt} = -0.07A$, where A is the amount of the element present at time t, in days.

(a) Find a function that satisfies the equation if the amount of the element present at $t = 0$ is 800 grams.

(b) After 20 days, how much of the 800 grams will remain? Round to the nearest gram.

(c) After how long will half of the original amount remain?