Name:

MATH 192 Exam 3 Hand portion

Show all work to receive credit. No calculators are permitted on this portion of the exam. It would be useful to hand this in after 10 minutes; it is suggested that you hand it in after 15 minutes; you must hand it in, and take the calculator portion, by 25 minutes into the period.

1. a) A company finds that the marginal cost, $C'$, in dollars, to make the $x$th unit of a product is $C'(x) = x^3 - 5$. Find the total cost function $C$ assuming that $C(0) = 100$.

(6)

$$C(x) = \int x^3 - 5 \, dx = \frac{x^4}{4} - 5x + c$$

$$C(0) = c = 100$$

$$C(x) = \frac{x^4}{4} - 5x + 100$$

b) Find the average value of $x^3 - x + 4$ over the interval $[0, 2]$; you need not simplify.

(8)

$$\text{average value} = \frac{1}{2} \int_0^2 x^3 - x + 4 \, dx$$

$$= \frac{1}{2} \left( \frac{x^4}{4} - \frac{x^2}{2} + 4x \right) \Big|_0^2$$

$$= \frac{1}{2} ( 4 - 2 + 8 )$$

$$= 5$$

(8) c) Determine whether the improper integral $\int_3^\infty \frac{1}{x^3}\,dx$ is convergent or divergent, and calculate its value if it is convergent.

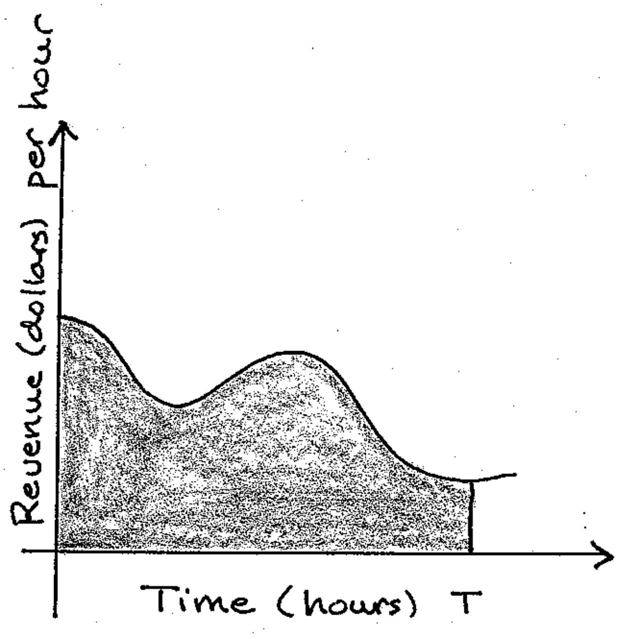
$$= \lim_{b\to\infty} \int_3^b x^{-3}\,dx$$

$$= \lim_{b\to\infty} \left.\frac{x^{-2}}{-2}\right|_3^b$$

$$= \lim_{b\to\infty} \left(-\frac{1}{2b^2} + \frac{1}{2\cdot 9}\right) = \frac{1}{18}$$

converges

(8) 2. Explain what the shaded area represents.




Amount of revenue over the T hour period in dollars

Name:

MATH 192 Exam 3 Calculator portion

Show all work to receive credit. Calculators are permitted on this portion of the exam, but please try to make clear what was done by calculator and be careful to follow directions.

(12) 3. For the demand function given by $D(x) = 16800 + 40x - 20x^2$ for $0 \le x \le 30$,

a) find the elasticity at $x = \$20$, state whether demand is elastic or inelastic, and state what that predicts for the effect on revenue of a small increase in price;

$$E = \frac{-x\,D'(x)}{D(x)} = \frac{-x\,(40 - 40x)}{16800 + 40x - 20x^2}$$

$$E(20) = \frac{-20\,(40 - 40(20))}{16800 + 40(20) - 20(20^2)}$$

$$= \frac{19}{12} > 1 \quad \text{elastic}$$

A small increase in price will lower the revenue

(8) b) find, from the point of view of elasticity of demand, the point at which revenue is a maximum.

$$E = 1 = \frac{-40x + 40x^2}{16800 + 40x - 20x^2}$$

$$16800 + 40x - 20x^2 = -40x + 40x^2$$

$$0 = 60x^2 - 80x - 16800$$

$$0 = 3x^2 - 4x - 840$$

$$x = \frac{+4 \pm \sqrt{16 + 12(840)}}{6} = 17.41$$

(plus a negative value that is irrelevant)

If the price is set at \$17.41 the revenue is at a maximum.

4. Find the area of the region bounded by the graphs of the given quantities:
(10) $y = 2x^2 - 6x + 5$ and $y = x^2 + 6x - 15$.

we find the intersection points:

$$2x^2 - 6x + 5 = x^2 + 6x - 15$$

$$x^2 - 12x + 20 = 0 \qquad x = 2, 10$$

(solved with calculator)

$$\text{Area} = \left| \int_2^{10} 2x^2 - 6x + 5 - (x^2 + 6x - 15)\, dx \right|$$

$$= \left| \int_2^{10} x^2 - 12x + 20\, dx \right| = \left| -\frac{256}{3} \right| = \frac{256}{3}$$

(10) 5. A company finds that the marginal cost, in dollars, of laying railroad track is given by $C'(x) = x^2 - 10x + 150$ for $0 \leq x \leq 100$, where $x$ is measured in hundreds of feet. Use four subintervals over $[0, 20]$ and left endpoints of each subinterval to approximate the total cost of laying 2000 feet of track. [Note that you are required to do a subinterval approximation; other calculational approaches will not receive credit.]

$$\text{cost} \cong C'(0) \cdot 5 + C'(5) \cdot 5 + C'(10) \cdot 5 + C'(15)\, 5$$

$$= 5\,(150 + 125 + 150 + 225)$$

$$= 3250 \text{ dollars}$$

(10) 6. If $D(x) = (x - 10)^2$ is the price, in dollars per unit, that customers are willing to pay for $x$ units of an item ($x \leq 10$), and $S(x) = x^2$ is the price, in dollars per unit, that producers are willing to accept for $x$ units ($x \leq 10$), find the consumer and producer surplus at the equilibrium point, labeling clearly which is which.

equilibrium point: $(x-10)^2 = x^2$

$x^2 - 20x + 100 = x^2$

$10(-2x+10) = 0 \qquad x = 10/2 = 5$

$S(5) = 25$

equilibrium point is $(5, 25)$

Consumer surplus $= \int_0^5 (x-10)^2\,dx - 5 \cdot 25$

$= \frac{500}{3} \cong 166.67$ dollars

producer surplus $= 5 \cdot 25 - \int_0^5 x^2\,dx$

$= \frac{250}{3} \cong 83.33$ dollars

(7) 7. a) What should the rate of a continuous money flow, at constant rate for a period of 15 years at 5% compounded continuously, be so that the future value will be $20,000?

$$\text{Future value} = e^{(.05)(15)} \int_0^{15} P e^{-.05t}\,dt = e^{.75} P \left. \frac{e^{-.05t}}{-.05} \right|_0^{15}$$

$$= P e^{.75} \left( \frac{e^{-.75}}{-.05} - \frac{1}{-.05} \right) = P \frac{e^{.75}-1}{.05} = 20{,}000$$

$$P = \frac{.05}{e^{.75}-1}\, 20{,}000 = 895.26 \quad \$/\text{year}$$

(7) b) What is the present value of the money flow in part a), still using 5% compounded continuously? [If you did not solve part a), assume that the rate of flow is $2000 per year, and indicate that you have done so.]

$$\text{Present value} = 20000\, e^{-.05(15)}$$

$$= 9447.33 \quad \$$$

(6) c) Now suppose the flow rate of part a) is continued perpetually; what is its present value, still using 5% compounded continuously? [Again, if you did not solve part a), assume that the rate of flow is $2000 per year, and indicate that you have done so.]

$$\text{present value} = \frac{P}{k} = \frac{895.26}{.05} = 17905.20 \ \$$$