Name:

MATH 192 Exam 2

Show all work to receive credit. Calculators are permitted on this exam, but please try to make clear what was done by calculator and be careful to follow directions.

1. Compute

(6) a) $\frac{dy}{dx}$ if $y^2x^2 - 3xy + \frac{4}{x} = 2$;

$$2y\frac{dy}{dx}x^2 + y^2 2x - 3y - 3x\frac{dy}{dx} - \frac{4}{x^2} = 0$$

$$\frac{dy}{dx}(2yx^2 - 3x) = -2xy^2 + 3y + \frac{4}{x^2}$$

$$\frac{dy}{dx} = \frac{-2xy^2 + 3y + \frac{4}{x^2}}{2yx^2 - 3x}$$

(4) b) The equation of the tangent line to the graph from part a) at the point $(1, 1)$;

$$m = \left.\frac{dy}{dx}\right|_{(1,1)} = \frac{-2 + 3 + 4}{2 - 3} = -5$$

$$y - 1 = -5(x-1)$$

$$y = -5x + 5 + 1$$

$$y = -5x + 6$$

(8) 1. (continued, Compute) c) $\frac{dy}{dx}$ if $e^{x^2y} = 3 - \ln y$;

$$e^{x^2y}\left(2xy + x^2\frac{dy}{dx}\right) = -\frac{1}{y}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(x^2e^{x^2y} + \frac{1}{y}\right) = -2xy\,e^{x^2y}$$

$$\frac{dy}{dx} = -\frac{2xy\,e^{x^2y}}{x^2e^{x^2y} + \frac{1}{y}}$$

(5) d) How much should be invested now to yield a payment of \$1000 three years from now if the interest rate is 5% yearly, compounded continuously? (That is, what is the present value of such a payment?)

$$1000 = P_0\,e^{.05(3)}$$

$$1000\,e^{-.15} = P_0$$

$$P_0 = 860.71 \quad \text{dollars}$$

(16) 2. Graph the following, justifying your result by methods of calculus; label all local maxima and minima and points of inflection: $y = 4x^4 - 6x^2 + 3$.

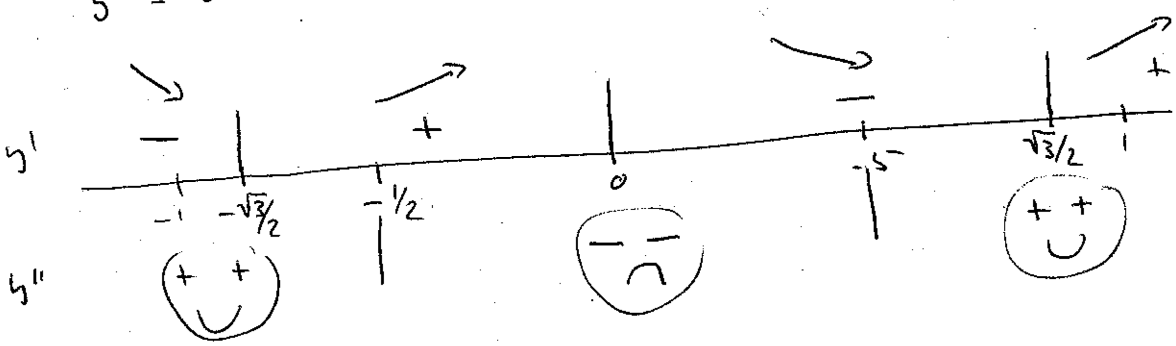
$$y' = 16x^3 - 12x = 4x(4x^2 - 3)$$

$$y'' = 48x^2 - 12 = 12(4x^2 - 1)$$

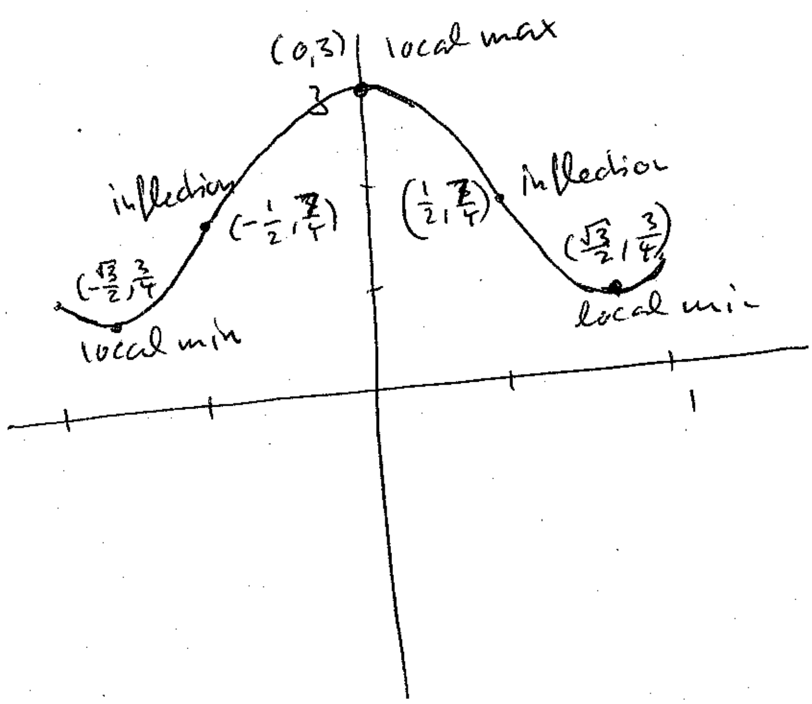
$$y' = 0 \Rightarrow 4x(4x^2 - 3) = 0 \Rightarrow x = 0 \text{ or } 4x^2 - 3 = 0$$

$$x^2 = 3/4 \quad \text{so} \quad x = \pm \frac{\sqrt{3}}{2} \cong \pm .87$$

$$y'' = 0 \Rightarrow 4x^2 - 1 = 0 \Rightarrow x^2 = 1/4 \Rightarrow x = \pm 1/2$$



x	y
-1	1
$-\sqrt{3}/2$	.75
$-1/2$	1.75
0	3
$1/2$	1.75
$\sqrt{3}/2$	.75
1	1



(12) 3. Find, by methods of calculus, the absolute maximum and minimum values of the function over the interval, and indicate the x-values at which they occur: $f(x) = 24 + 18x - 6x^2 - 2x^3$; $[-4, 1]$.

$$f'(x) = 18 - 12x - 6x^2 = 6(3 - 2x - x^2)$$
$$= 6(3 + x)(1 - x)$$

critical values: $-3, 1$

x	$f(x)$
-4	-16
-3	-30
1	34

absolute max is 34 at $x = 1$

absolute min is -30 at $x = -3$

(12) 4. A company determines that the revenue, in thousands of dollars, for each one thousand units is $r(x) = 9 - 2x$, where x is the number of thousands of units. The total cost (in thousands of dollars) to produce x thousand units is $C(x) = x^3 - 3x^2 + 4x + 1$. Determine the number of units to produce for maximum profit, justifying your answer by methods of calculus.

$$P(x) = x(9 - 2x) - (x^3 - 3x^2 + 4x + 1)$$
$$= 9x - 2x^2 - x^3 + 3x^2 - 4x - 1$$
$$= -x^3 + x^2 + 5x - 1$$

Maximize $P(x) = -x^3 + x^2 + 5x - 1$ for $x > 0$

$$P'(x) = -3x^2 + 2x + 5$$
$$P'(x) = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4 + 60}}{-6} = \frac{-2 \pm 8}{-6}$$
$$x = \frac{10}{6} = \frac{5}{3} \quad \text{or} \quad x = \frac{6}{-6} = -1$$

$$P''(x) = -6x + 2, \quad P''\left(\tfrac{5}{3}\right) = -8 < 0 \Rightarrow \text{max}$$

Hence $\frac{5}{3} 1000 \cong 1667$ units produce the maximum profit.

5. a) Find the differential dy if $y = e^{2x}$;

(5)

$$dy = 2e^{2x}\,dx$$

b) Find a decimal approximation of $e^{.02}$ using differentials; make your method clear. [Merely computing $e^{.02}$ with your calculator will not be worth credit.]

(8) $f(x) = e^{2x}$ $\quad f(.01) \cong f(0) + f'(0)(.01)$

$\Delta x = .01$

$$= e^0 + 2e^0(.01)$$

$$= 1 + .02$$

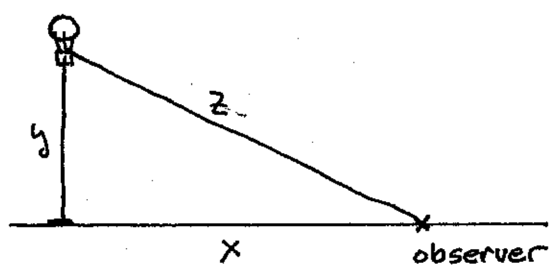
$$= 1.02$$

(you could have used $f(x) = e^x$ with $\Delta x = .02$)

(12) 6. A hot-air balloon is rising at a rate of 30 feet per minute. An observer on level ground due East of the balloon knows that the balloon is also being carried due West by the wind at a rate of 10 feet per minute. It is noon, so the shadow of the balloon is always directly underneath it. How fast is the distance between the observer and the balloon changing when the distance between them is 2000 feet and the shadow of the balloon is 1000 feet from the observer?

$$\frac{dy}{dt} = 30$$

$$\frac{dx}{dt} = 10$$



when $x = 1000$, $z = 2000$ then $y = \sqrt{z^2 - x^2} = \sqrt{2000^2 - 1000^2}$
$= \sqrt{3}\,1000$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

$$\frac{dz}{dt} = \frac{1000(10) + \sqrt{3}\,1000(30)}{2000} = \frac{10 + 30\sqrt{3}}{2}$$

$\cong$ 30.98 feet per minute.

7. You are offered two investments, one paying \$3000 ten years from now, and the other paying \$4000 fifteen years from now.

(4) a) Which is preferable from the standpoint of present value if the interest rate is 5% annually, compounded continuously?

$$P_1 e^{(.05)10} = 3000 \qquad P_1 = 3000\, e^{-.5} = 1819.59$$

$$P_2 e^{(.05)15} = 4000 \qquad P_2 = 4000\, e^{-.75} = 1889.47$$

You should take the fifteen year investment, it has a higher present value.
If you want to spend as little present money as possible, then take the 10 year option.

(4) b) When should the \$3000 be paid to make its present value equal to the present value of the \$4000 payment at fifteen years if the interest rate is 5% annually, compounded continuously?

$$1889.47\, e^{.05t} = 3000$$

$$e^{.05t} = \frac{3000}{1889.47}$$

$$.05t = \ln \frac{3000}{1889.47}$$

$$t = \frac{1}{.05} \ln \frac{3000}{1889.47}$$

$$= 9.25$$

It should be paid in 9 years and 3 months

(4) c) What must the interest rate be if the present values of the two payments, \$3000 ten years from now and \$4000 fifteen years from now, are equal?

$$\left.\begin{aligned} P_1 e^{p10} &= 3000 \\ P_1 e^{p15} &= 4000 \end{aligned}\right\} \quad P_1 = 3000\, e^{-10p} = 4000\, e^{-15p}$$

$$e^{5p} = \frac{4}{3} \implies 5p = \ln \frac{4}{3}, \quad p = \frac{1}{5} \ln \frac{4}{3} \approxeq .0575$$

The interest rate is approximately 5.75%