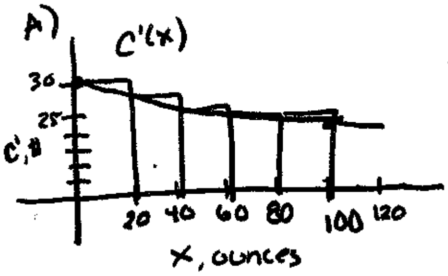
① $\Delta x = \frac{100-0}{5} = 20$

A)



B) $A = \Delta x\left[C'(0) + C'(20) + C'(40) + C'(60) + C'(80)\right]$

$= 20\left[30 + 28.2 + 26.8 + 25.8 + 25.2\right]$

$= \$2720$

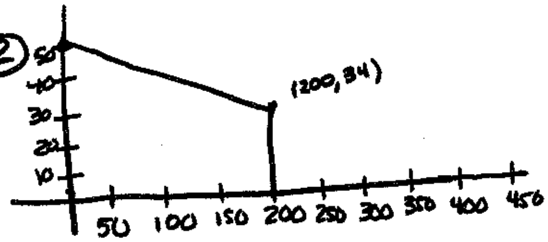
c) $C = \int(.0005x^2 - .1x + 30)\,dx = .0005\frac{x^3}{3} - \frac{.1x^2}{2} + 30x + C$

$C(0) = 1250$ [fixed costs)

$C = 1250$

$C = .000167x^3 - .05x^2 + 30x + 1250$

②



a. $A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(200)(50+34) = \underline{8400}$

b. $\int_0^{200}\left(\frac{-2}{25}x + 50\right)dx = \frac{-1}{25}x^2 + 50x\Big|_0^{200} = \underline{8400}$

③ $\sum_{i=1}^{4} C'(x_i)\Delta x = C'(0)\Delta x + C'(50)\Delta x + C'(100)\Delta x + C'(150)\Delta x$

$\Delta x = 50$

$= (500)(50) + (410)(50) + (340)(50) + (290)(50)$

$= \$77{,}000$

④ a. $\int 2e^{-5x}dx = \frac{2e^{-5x}}{-5} + C = \frac{-2}{5}e^{-5x} + C$

b. $\int\left(3t^2 - 4t + \frac{7}{t^4}\right)dt = t^3 - 2t^2 + \frac{7t^{-3}}{-3} + C = t^3 - 2t^2 - \frac{7}{3}t^{-3} + C$

c. $\int \frac{7e}{x}dx = \int(7e)x^{-1}dx = 7e \cdot \ln|x| + C$

d. $\int(2\pi^2 - x^e + e)dx = 2\pi^2 x - \frac{x^{e+1}}{e+1} + ex + C$

e. $f'(x) = \frac{4}{\sqrt{x}} = 4x^{-1/2}$ $f(1) = -5$

$\int 4x^{-1/2}dx = \frac{4x^{1/2}}{1/2} + C = 8x^{1/2} + C$

$f(1) = -5 \Rightarrow -5 = 8\sqrt{1} + C$

$-13 = C$

$\underline{f(x) = 8\sqrt{x} - 13}$

⑤

a. $\int_2^6 (7\sqrt{x}-2)\,dx = \int_2^6 7x^{1/2}-2\,dx = \dfrac{7x^{3/2}}{3/2} - 2x \Big|_2^6 = \dfrac{14}{3}x^{3/2} - 2x \Big|_2^6$

$= \left(\frac{14}{3}\cdot 6^{3/2} - 12\right) - \left(\frac{14}{3}\cdot 2^{3/2} - 4\right)$

$\boxed{= 47.386}$

b. $\int_{-3}^{2} 8x^3 + 5e^{-x}\,dx = 2x^4 - 5e^{-x}\Big|_{-3}^{2} = \left(2\cdot 16 - 5e^{-2}\right) - \left(2(-3)^4 - 5e^{3}\right)$

$= \left(32 - 5e^{-2}\right) - \left(162 - 5e^{3}\right)$

$= -130 - 5e^{-2} + 5e^{3} = \boxed{-30.249}$

c. $\int_1^5 \frac{5}{x} + \frac{5}{x^4}\,dx = 5\ln|x| + \dfrac{5x^{-3}}{-3}\Big|_1^5 = \left(5\ln 5 - \frac{5}{3\cdot 125}\right) - \left(5\cdot\ln 1 - \frac{5}{3}\right)$

$= 5\ln 5 - \frac{1}{75} + \frac{5}{3}$

$= \boxed{9.7005}$

⑥ $a = -32\ ft/s^2$

$V(t) = \int a(t)\,dt = \int -32\,dt = -32t + C$

$V_0 = 0 \quad 0 = -32(0) + C$

$0 = C$

$s(t) = \int v(t)\,dt = \int -32t\,dt = -16t^2 + C \qquad s_0 = 100$

$100 = -16(0)^2 + C$

$100 = C$

$s(t) = -16t^2 + 100$

Ball hits the ground when $s(t) = 0$ $\quad -16t^2 + 100 = 0$

$-16t^2 = -100$

$t^2 = \frac{100}{16}$

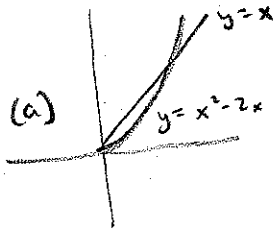
$t = \frac{10}{4} = \underline{2.5\ \text{SECONDS}}$

7) $\displaystyle\int_0^{50} (\sqrt{x} + 10x)\,dx = \int_0^{50} (x^{1/2} + 10x)\,dx = \frac{2}{3}x^{3/2} + \frac{10x^2}{2}\Big|_0^{50}$

$$= \left(\frac{2}{3}\,50^{3/2} + 5\cdot 50^2\right) - (0+0)$$

$$= \frac{100}{3}\sqrt{50} + 125{,}000 \quad \text{(dollars)}.$$

8) (a)



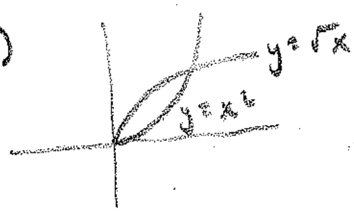
$$x^2 - 2x = x$$
$$x^2 = 3x$$
$$x = 0 \text{ or } 3$$

$$\int_0^3 (x - (x^2 - 2x))\,dx$$

$$= \int_0^3 (3x - x^2)\,dx$$

$$= \frac{3x^2}{2} - \frac{x^3}{3}\Big|_0^3 = \left(\frac{3\cdot 3^2}{2} - \frac{3^3}{3}\right) - (0-0) = 9/2$$

(b)



$$x^2 = \sqrt{x}$$
$$x^4 = x$$
$$x^4 - x = 0$$
$$x(x^3 - 1) = 0$$
$$x = 0, 1$$

$$\int_0^1 (\sqrt{x} - x^2)\,dx = \frac{2}{3}x^{3/2} - \frac{1}{3}x^3\Big|_0^1$$

$$= (2/3 - 1/3) - (0-0)$$

$$= 1/3$$

9) (a) $\displaystyle\frac{1}{2-(-1)}\int_{-1}^{-3} (3x^6 + 1/x)\,dx = \frac{1}{3}\left[\frac{3x^7}{7} + \ln|x|\right]\Bigg|_{-1}^{-3}$

$$= \frac{1}{3}\left(\left(\frac{3(-3)^7}{7} + \ln(3)\right) - \left(\frac{3(-1)^7}{7} + \underbrace{\ln(1)}_{0}\right)\right)$$

$$= \frac{1}{3}\left[-\frac{3^8}{7} + \ln(3) + \frac{3}{7}\right]$$

9) (b) $\frac{1}{7-3}\int_3^7 2e^{-2x}\,dx = \frac{1}{4}\left[\frac{2e^{-2x}}{-2}\right]_3^7$

$= \frac{1}{4}\left[\left(-e^{-2(7)}\right)-\left(-e^{-2(3)}\right)\right] = \frac{1}{4}\left(-e^{-14}+e^{-6}\right)$

(c) $\frac{1}{2-1}\int_1^2 \frac{2}{x^4}\,dx = \int_1^2 2x^{-4}\,dx = \left.\frac{2x^{-3}}{-3}\right|_1^2 = \left(\frac{2\cdot 2^{-3}}{-3}\right) - \left(\frac{2\cdot 1^{-3}}{-3}\right)$

$= \frac{2}{2^3\cdot 3} + \frac{2}{3} = \frac{1}{12} + \frac{2}{3} = \frac{9}{12} = \frac{3}{4}$

10)

(a) $D(x) = S(x)$

$(x-10)^2 = x^2$

$x^2 - 20x + 100 = x^2$

$20x = 100$

$\underline{x = \boxed{5}}$ units

(b) $\int_0^5 D(x)\,dx - 5\,D(5)$

$= \int_0^5 (x^2 - 20x + 100)\,dx \quad - 5\left[(5-10)^2\right]$

$= \left.\frac{x^3}{3} - 10x^2 + 100x\right|_0^5 - 5(25)$

$= \left(\frac{5^3}{3} - 10\cdot 5^2 + 100\cdot 5\right) - (0) - 125$

$= \frac{125}{3} - 250 + 500 \quad - 125$

$= \frac{500}{3} \quad = \$166.67$

10) (c) $5S(5) - \int_0^5 S(x)\,dx$

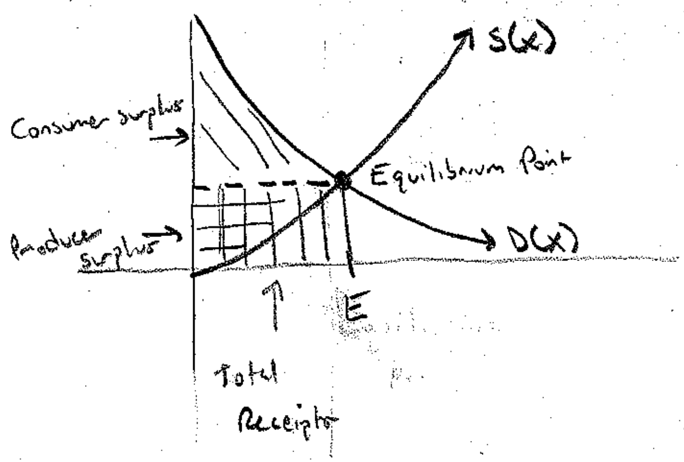
$= 5(5^2) - \int_0^5 x^2\,dx$

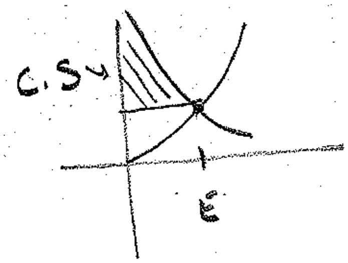
$= 125 - \frac{x^3}{3}\Big|_0^5$

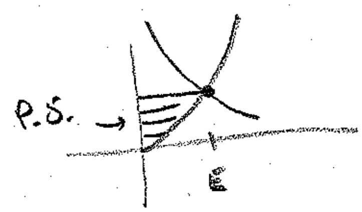
$= 125 - \left(\frac{125}{3} - 0\right)$

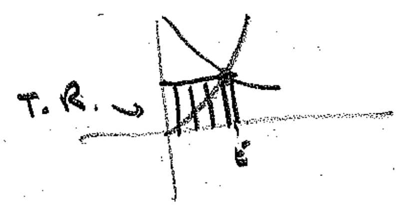
$= \frac{250}{3} = \$83.33$

11)









(a) Equilibrium Point:
When supply equals demand.
Gives price at which purchases and sales actually occurs

(b) Consumer Surplus:
Utility the consumer receives beyond the total receipts, that is, amount he or she paid.

(c) Producer Surplus:
Surplus over cost. Benefit Producer recieves when supplying more units at a higher cost than at a price at which fewer units are supplied.

(d) Total receipts: Number of Items Purchased $*$ Price per item. $\left[\begin{array}{l} xS(x) \\ \text{or } xD(x) \end{array}\right]$

12) (a) $10{,}000\, e^{.07(10)} = \$20,137.53$

(b) $\int_0^{10} 1500\, e^{.07t}\, dt = \frac{1500}{.07} e^{.07t}\Big|_0^{10} = \frac{1500}{.07}\left(e^{.7}-1\right)$

$= \$21,723.27$

Option (b) has a higher future value.

13)

a) $P(t) = P_0 e^{kt} = 25000\, e^{0.05t}$

$P(15) \approx \$ 52925$

b) $\int_0^T R(t) e^{-k(T-t)}\, dt = \int_0^{15} 1100\, e^{-0.05(15-t)}\, dt$

$= \int_0^{15} (1100)(e^{-0.05 \cdot 15}) e^{0.05t}\, dt$

$= (1100)\, e^{-.75} \int_0^{15} e^{0.05t}\, dt$

$= 519.60 \left[\frac{e^{0.05t}}{0.05} \Big|_0^{15} \right]$

$= 519.60 \left[20 \left(e^{0.75} - 1 \right) \right]$

$= \$ 11607.90$

So (a) is better

4] $P(t) = P_0 e^{kt}$. Letting 2005 $\leftrightarrow t=0$, and $P(t)$ be measured in billions of metric tons

$P(t)$ becomes $P(t) = 1.23 e^{0.03t}$.

we want

$$\int_0^{11} 1.23 e^{0.03t}\, dt = 1.23 \int_0^{11} e^{0.03t}\, dt$$

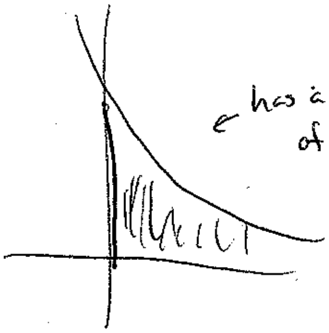
$$= 1.23 \left[\left. \frac{e^{0.03t}}{0.03} \right|_0^{11} \right]$$

$$= 1.23 \left[13.023 \right]$$

$$= 16.02 \text{ billion metric tons}$$

15)

a) $$\int_0^\infty e^{-4x}\,dx = \lim_{b\to\infty} \int_0^b e^{-4x}\,dx$$



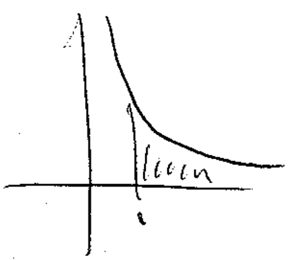
← has a chance of converging

$$= \lim_{b\to\infty} \left.\frac{e^{-4x}}{-4}\right|_0^b$$

$$= \lim_{b\to\infty} \frac{e^{-4b}}{-4} - \frac{e^0}{-4}$$

$$= \lim_{b\to\infty} \frac{1}{4} - \frac{e^{-4b}}{4}$$

$$= \frac{1}{4}$$

b) 

has a chance of converging

$$\int_1^\infty \frac{1}{x^{1/2}}\,dx = \int_1^\infty x^{-\frac{1}{2}}\,dx$$

$$= \lim_{b\to\infty} \int_1^b x^{-1/2}\,dx$$

$$= \lim_{b\to\infty} \left.\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right|_1^b$$

$$= \lim_{b\to\infty} 2\sqrt{b} - 2$$

$$= +\infty$$

16)

$$\int_1^\infty 2000\, x^{-3.2}\, dx = \lim_{b\to\infty} \int_1^b 2000\, x^{-3.2}\, dx$$

$$= \lim_{b\to\infty} \left. \frac{2000\, x^{-2.2}}{-2.2} \right|_1^b$$

$$= \lim_{b\to\infty} \frac{2000\, b^{-2.2}}{-2.2} - \frac{2000}{-2.2}$$

$$= \lim_{b\to\infty} 909.09 - \frac{909.09}{b^{2.2}}$$

$$= \$909.09$$

17]

a) $q = 200 - 100(.75)$

$= 125$

b) $E(x) = -\dfrac{x\,D'(x)}{D(x)}$

$= -\dfrac{x(-100)}{200-100x}$

$= \dfrac{100x}{200-100x}$

c) $E(0.5) = 0.33 < 1 \Rightarrow$ inelastic

$E(1.5) = 3 > 1 \Rightarrow$ elastic

d) $1 = \dfrac{100x}{200-100x} \Rightarrow$ $200 - 100x = 100x$

$200 = 200x$

$x = 1$

e) total revenue is (number sold) $\cdot$ (price) $= x\,D(x)$

$= 200x - 100x^2$

f) Revenue is maximized when Elasticity $= 1$ so at $x = 1$, in this case.