Name:

MATH 192 Exam 3 Hand Portion

Show all work to receive credit. Calculators are not permitted on this portion of the exam. You are encouraged to turn in this portion of the exam and receive the calculator portion after 15 minutes; you are strongly encouraged to make the switch after 20 minutes; you are required to turn in this portion of the exam, and receive the calculator portion, no later than 25 minutes into the examination.

1. Compute as requested:

a) $\int \frac{4}{\sqrt[5]{x}} + e^{2x}\,dx$; $= 4\int x^{-\frac{1}{5}}\,dx + \int e^{2x}\,dx$

$$= 4\,\frac{x^{4/5}}{4/5} + \frac{1}{2}e^{2x} + C$$

$$= 5x^{4/5} + \frac{1}{2}e^{2x} + C$$

b) Find f such that $f'(x) = 8x^2 - 3x + 2$ and $f(0) = 2$;

$$f(x) = \int 8x^2 - 3x + 2\,dx = \frac{8}{3}x^3 - \frac{3}{2}x^2 + 2x + C$$

$$2 = f(0) = C \Rightarrow C = 2$$

$$f(x) = \frac{8}{3}x^3 - \frac{3}{2}x^2 + 2x + 2$$

c) $\int_1^8 \sqrt[3]{x}\,dx$; $= \int_1^8 x^{1/3}\,dx = \frac{3}{4}x^{4/3}\Big|_1^8$

$$= \frac{3}{4}\left(\left(\sqrt[3]{8}\right)^4 - 1\right)$$

$$= \frac{3}{4}(16 - 1) = \frac{45}{4}$$

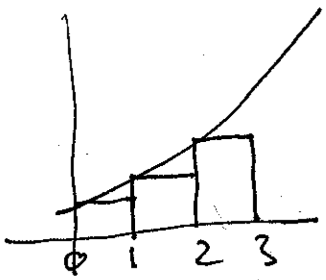
d) $\int_2^\infty 7x^{-3}\,dx$. $= \lim_{b\to\infty} \int_2^b 7x^{-3}\,dx = \lim_{b\to\infty} -\frac{7}{2}x^{-2}\Big|_2^b$

$$= \lim_{b\to\infty}\left(-\frac{7}{2b^2} + \frac{7}{8}\right) = \frac{7}{8}$$, converges

2. a) Approximate the area under the curve $y = x^2 + 8x + 4$ between $x = 0$ and $x = 3$ using a three rectangle approximation with left hand endpoints. DO NOT SIMPLIFY YOUR ANSWER.

Area $\approx y(0) + y(1) + y(2) = 4 + 13 + (4 + 16 + 4)$

$= 41$



b) Suppose the curve of part a) approximates y, the number of customers per hour at a certain store, in terms of x, the number of hours since opening. Interpret the area you approximated in part a) in practical terms other than "area." Be sure to include units.

Area is number of customers that visited this store during the first three hours since opening.

Name:

MATH 192 Exam 3 Calculator Portion

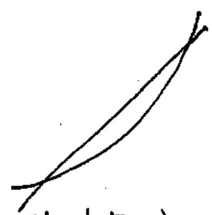
Show all work to receive credit. Calculators are permitted on this portion of the exam, but please try to make clear what is done by calculator.

3. a) Find the area enclosed by the curves $y = x^2$ and $y = 3x - 2$.

Intersection: $x^2 = 3x - 2$

$x^2 - 3x + 2 = 0$ $\qquad x = \dfrac{3 \pm \sqrt{9-8}}{2}$

$x = 1, 2$

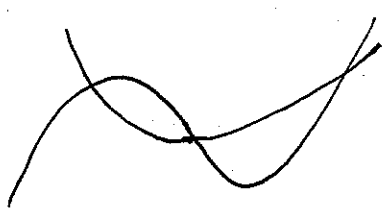


(calculator)

$$\text{Area} = \int_1^2 3x - 2 - x^2 \, dx = 1/6$$

with calculator

b) Find the area enclosed by the curves $y = x^3 + 4x^2 - 3x - 1$ and $y = 2x^2 + 2x + 5$.



(Calculator)

intersection:

$x^3 + 4x^2 - 3x - 1 = 2x^2 + 2x + 5$

$x^3 + 2x^2 - 5x - 6 = 0$

$x = -3, -1, 2$ (with calculator)

$$\text{Area} = \int_{-3}^{-1} x^3 + 4x^2 - 3x - 1 - (2x^2 + 2x + 5) \, dx + \int_{-1}^{2} 2x^2 + 2x + 5 - (x^3 + 4x^2 - 3x - 1) \, dx$$

$$= \int_{-3}^{-1} x^3 + 2x^2 - 5x - 6 \, dx + \int_{-1}^{2} -x^3 - 2x^2 + 5x + 6 \, dx$$

$$= \frac{253}{12}$$ (with calculator)

4. Suppose that $D(x) = x^2 - 12x + 36$ is the price, in dollars per unit, that customers are willing to pay for x units of an item, and $S(x) = x^2 + 3x + 6$ is the price, in dollars per unit, that producers are willing to accept for x units.

a) Find the equilibrium point.

$$D(x) = S(x)$$

$$x^2 - 12x + 36 = x^2 + 3x + 6$$

$$-15x + 30 = 0$$

$$x - 2 = 0 \quad , x = 2$$

$$D(2) = 4 - 24 + 36 = 16$$

equilibrium point:

$(2, 16)$

b) Find the consumer surplus at the equilibrium point.

$$\text{Consumer surplus} = \int_0^2 D(x)\,dx - 2(16)$$

$$= \int_0^2 x^2 - 12x + 36\,dx - 32$$

$$= \frac{56}{3} \text{ dollars}$$

(with calculator)

c) Find the producer surplus at the equilibrium point.

$$\text{Producer surplus} = 2(16) - \int_0^2 S(x)\,dx$$

$$= 32 - \int_0^2 x^2 + 3x + 6\,dx$$

$$= \frac{34}{3} \text{ dollars}$$

(with calculator)

5. a) You are offered an investment yielding a continuous money flow at the rate of $R_1(t) = 1000 + 500t$ dollars per year from now until 10 years into the future, with interest at 8% and compounded continuously. Find the accumulated present value of this investment.

$$\text{Present value} = \int_0^{10} R_1(t)\, e^{-.08t}\, dt$$

$$= \int_0^{10} (1000 + 500t)\, e^{-.08t}\, dt$$

$$= 21\,821.50 \quad \text{dollars}$$

(with calculator)

b) A second investment becomes available, which is a perpetual continuous money flow of \$2000 dollars per year; again, the interest rate is 8% and compounded continuously. What is the present value of this investment? Which investment should you prefer on present value grounds, and why?

$$\text{Present value} = \int_0^{\infty} 2000\, e^{-.08t}\, dt$$

$$= 2000 \lim_{b \to \infty} \int_0^{b} e^{-.08t}\, dt$$

$$= 2000 \lim_{b \to \infty} \left(\frac{e^{-.08t}}{-.08} \Big|_0^b \right)$$

$$= 2000 \lim_{b \to \infty} \left(-\frac{e^{-.08b}}{.08} + \frac{1}{.08} \right)$$

$$= \frac{2000}{.08} = 25\,000 \quad \text{dollars}$$

The latter investment has a bigger present value. So it is the more valuable one and we take it.

5. (continued) c) If the investment of part b) were changed to run from now until T years into the future, what value of T will make the accumulated present value of the investment \$20,000 assuming the constant continuous money flow stays the same (that is \$2000 per year) and the interest rate remains 8% compounded continuously?

$$\int_0^T 2000\, e^{-.08t}\, dt = 20{,}000$$

$$2000 \left. \frac{e^{-.08t}}{-.08} \right|_0^T = 20{,}000$$

$$2000 \left(\frac{e^{-.08T}}{-.08} - \frac{1}{-.08} \right) = 20{,}000$$

$$\frac{2000}{.08} \left(1 - e^{-.08T} \right) = 20{,}000$$

$$1 - e^{-.08T} = \frac{.08}{2000} 20{,}000 = .8$$

$$.2 = e^{-.08T}$$

$$\ln(.2) = -.08T$$

$$T = -\frac{\ln(.2)}{.08}$$

$$T \approx 20.12 \text{ years}$$

5. A computer game company estimates that demand D for a new game (in number of games) is a function of the price x in dollars, with expression $D(x) = -15x^2 - 90x + 146025$.

a) Compute the elasticity of demand at a price of $35 per game. Is demand elastic or inelastic at this price? Interpret the answer in practical terms for the company.

$$E = -\frac{x\ D'(x)}{D(x)} = -\frac{x(-30x - 90)}{-15x^2 - 90x + 146025}$$

$$E(35) = \frac{133}{415} < 1 \quad \text{(with calculator)}$$

The demand is inelastic.

A price increase will increase the revenue.

b) Compute, from an elasticity of demand approach, the price for maximum revenue to the company.

Maximum revenue is obtained if $E = 1$

$$1 = \frac{30x^2 + 90x}{-15x^2 - 90x + 146025}$$

$$-15x^2 - 90x + 146025 = 30x^2 + 90x$$

$$-45x^2 - 180x + 146025 = 0$$

$$x^2 + 4x - 3245 = 0$$

$$x = \frac{-4 \pm \sqrt{16 + 4(3245)}}{2} = 55$$

Maximum revenue is obtained if the price is set at \$ 55 per game.