Name:

MATH 192 Exam 1

Show all work to receive credit. No calculators are permitted on this exam.

1. Compute but DO NOT SIMPLIFY

a) $f'(x)$ if $f(x) = \frac{1}{x^2} - 3x^4$; $= x^{-2} - 3x^4$

(4)

$f'(x) = -2x^{-3} - 12x^3$

b) $f''(x)$ if f is the function in part a);

(4)

$f''(x) = 6x^{-4} - 36x^2$

(6) c) $\frac{d}{dx}\left(\frac{x^2+3}{x^6+2x^3+1}\right)$; $= \dfrac{2x(x^6+2x^3+1) - (x^2+3)(6x^5+6x^2)}{(x^6+2x^3+1)^2}$

(6) d) $\frac{dy}{dx}$ if $y = (x^3+2)^{27/2}$.

$\frac{dy}{dx} = \frac{27}{2}(x^3+2)^{\frac{25}{2}}(3x^2)$

2. Let the cost in dollars of producing x goods be $c(x) = 200 + x^{1/2}(100x^{1/4} + 20)$.

a) Find the average cost, $A(x)$, of producing x goods (you need not simplify).

(3)

$$A(x) = \frac{c(x)}{x} = \frac{200 + x^{1/2}(100x^{1/4} + 20)}{x}$$

(7) b) Compute (you need not simplify), give the units for, and explain the meaning in practical terms of,

$c(5)$ $= 200 + 5^{1/2}(100(5^{1/4}) + 20$

This is the total cost to produce 5 goods.
It is in "dollars"

$c'(5)$ $c(x) = 200 + 100x^{3/4} + 20x^{1/2}$

$c'(x) = 100\frac{3}{4}x^{-1/4} + 20\frac{1}{2}x^{-1/2}$

$c'(5) = 75(5^{-1/4}) + 10(5^{-1/2})$

This is the instantaneous rate of change of the cost at 5 goods. (This is approximately the cost to produce the 5th good.)

It is in "dollars per good."

$A(5)$

$$A(5) = \frac{200 + 5^{1/2}(100(5^{1/4}) + 20)}{5}$$

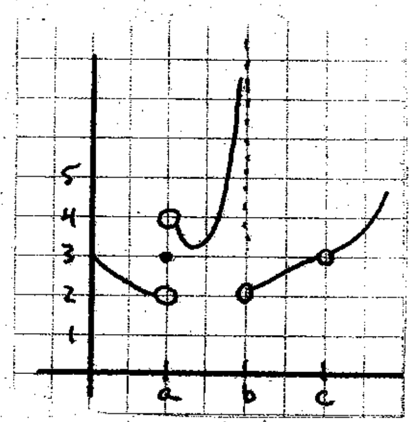
It is the average production cost per good if 5 goods are produced.

The units are "dollars per good".

3. Consider the graph of $y = f(x)$ shown below. Find, and record in the table, the values of the limits, the values of the left hand limits, and whether continuity holds (yes/no) for f at the points a through c. (If a limit or left hand limit does not exist, write DNE.)

(1 each)

z	$\lim_{x\to z} f(x)$	$\lim_{x\to z^-} f(x)$	continuous at z?
a	DNE	2	No
b	DNE	∞	No
c	3	3	No



4. a) Find a simplified form for the difference quotient for $f(x) = \frac{5}{x}$.

(6)

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{5}{x+h} - \frac{5}{x}}{h} = \frac{\frac{5x - 5(x+h)}{x(x+h)}}{h}$$

$$= \frac{5x - 5x - 5h}{h\,x\,(x+h)} = \frac{-5h}{h\,x\,(x+h)} = -\frac{5}{x(x+h)}$$

b) Find the equation of the tangent line to the graph of the function in part a) at $x = 3$.

(6) $f'(x) = -\frac{5}{x^2}$. Slope $= m = f'(3) = -\frac{5}{9}$

When $x = 3$, then $y = f(3) = \frac{5}{3}$, so $(3, \frac{5}{3})$ is a point on the line.

$$y - \frac{5}{3} = -\frac{5}{9}(x-3) = -\frac{5}{9}x + \frac{5}{3}$$

$$y = -\frac{5}{9}x + \frac{10}{3}$$

c) Find all points at which the tangent line to the graph of the function in part a) has slope -2.

(7) $f'(x) = -\frac{5}{x^2} = -2$

so $x^2 = \frac{5}{2}$ and thus $x = \pm\sqrt{5/2}$

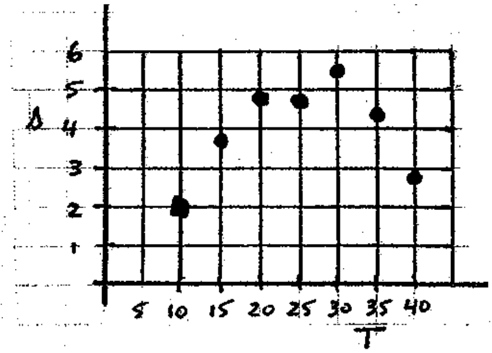
If $x = \sqrt{5/2}$ then $y = f(\sqrt{5/2}) = \frac{\sqrt{2}\,5}{\sqrt{5}} = \sqrt{10}$

If $x = -\sqrt{5/2}$ then $y = f(-\sqrt{5/2}) = -\frac{\sqrt{2}\,5}{\sqrt{5}} = -\sqrt{10}$

Hence the points are $(\sqrt{\frac{5}{2}}, \sqrt{10})$ and $(-\sqrt{\frac{5}{2}}, -\sqrt{10})$.

5. Consider the table of data showing grams of converted sugar s in a particular solution with s as a function of temperature T (in degrees Centigrade).

T (degrees C)	s (grams)
10°	2
15°	3.8
20°	4.9
25°	4.8
30°	5.5
35°	4.3
40°	2.9



(2) a) Give a rough plot of these data on the grid provided.

(4) b) For these data, would you prefer a linear fit, a quadratic fit with $a > 0$, a quadratic fit with $a < 0$, or a polynomial (neither quadratic nor linear) fit?

A quadratic fit with $a < 0$ in $s = aT^2 + bT + c$.

(7) c) Without worrying about your answer to b), construct a linear fit using the data points corresponding to $T = 10°$ and $T = 30°$.

Need a line through the points $(10, 2)$ and $(30, 5.5)$

slope $m = \frac{5.5 - 2}{30 - 10} = \frac{3.5}{20} = \frac{7}{40}$

$s - 2 = \frac{7}{40}(T - 10) = \frac{7}{40}T - \frac{7}{4}$

$s = \frac{7}{40}T - \frac{7}{4} + 2 = \frac{7}{40}T + \frac{1}{4}$

$s = \frac{7}{40}T + \frac{1}{4}$

(5) d) Use the result of part c) to predict the grams of converted sugar in a solution at temperature 22° (you need not simplify).

$s(22) = \frac{7}{40} 22 + \frac{1}{4}$ grams

6. Suppose demand q for a certain product (in hundreds) is a function of price x in dollars according to $q = (x-5)^2$.

(5) a) Find the average rate of change of demand with respect to price between a price of \$2 and a price of \$5, giving units.

$$\frac{q(5) - q(2)}{5-2} = \frac{0 - 9}{3} = -3 \quad \text{hundreds}/\$$$

(6) b) Find the instantaneous rate of change of demand with respect to price at the price of \$2, giving units.

$$q' = 2(x-5)$$

$$q'(2) = 2(-3) = -6 \quad \text{hundreds}/\$$$

(7) c) Suppose the price x is a function of time t (in years since 2000) given by $x = \frac{t^2}{16} + \frac{t}{4} + 2$. Find $\frac{dq}{dt}$ in the year 2004.

$$\frac{dq}{dt} = \frac{dq}{dx}\frac{dx}{dt} = 2(x-5)\left(\frac{t}{8} + \frac{1}{4}\right) \qquad t=4 \Rightarrow x = 1+1+2 = 4$$

$$\left.\frac{dq}{dt}\right|_{t=4} = 2(-1)\left(\frac{1}{2} + \frac{1}{4}\right) = -2\,\frac{3}{4} = -\frac{3}{2} \quad \text{hundreds/year}$$

(6) d) (Ignore part c) Suppose that along with the demand function q above there is a supply function $s = 2x^2 - 12x + 17$ (again in hundreds of units). Find the equilibrium point for this pair of supply and demand functions.

$$q(x) = s(x)$$

$$(x-5)^2 = 2x^2 - 12x + 17$$

$$x^2 - 10x + 25 = 2x^2 - 12x + 17$$

$$0 = x^2 - 2x - 8$$

$$0 = (x-4)(x+2)$$

$x = 4$ or $x = -2$

x cannot be negative, hence $x = 4$.

$x = 4 \Rightarrow q = (4-5)^2 = 1$

$(4, 1)$ is the equilibrium point.