The Structure of the Consecutive Pattern Poset

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Joint work with:
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Cornell Discrete Geometry and Combinatorics Seminar

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Outline

- Permutation patterns: classical and consecutive
- Consecutive pattern poset
- Results
- Open problems

Classical patterns

Definition. An occurrence of a permutation σ as a pattern in a permutation τ is a subsequence of τ whose letters are in the same relative order as those in σ.

Example. 231 occurs in twice in 416325: 416325 and 416325.

Example. An inversion in τ is equivalent to an occurrence of 21, e.g. 1423 and 1423.

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- Huge area of study in the last three decades.
- Most work is enumerative, esp. counting the number of permutations that avoid a given pattern.
- Knuth (1975), Rogers (1978): For any permutation $\sigma \in S_3$, the number of permutations in S_n avoiding σ is C_n.
- Open: closed formula for number avoiding 1324.

Consecutive patterns

Our focus:

Definition. An occurrence of a consecutive pattern σ in a permutation τ is a subsequence of adjacent letters of τ in the same relative order as those in σ.

Examples.

- 123 occurs twice in 7245136: 7245136 and 7245136.
- 416325 avoids the consecutive pattern 231.
- A descent is an occurrence of the consecutive pattern 21, e.g 4132 and 4132.
- A peak is an occurrence of 132 or 231, e.g., 13415.
- A permutation is alternating (up-down or down-up) iff it avoids 123 and 321 as consecutive patterns.

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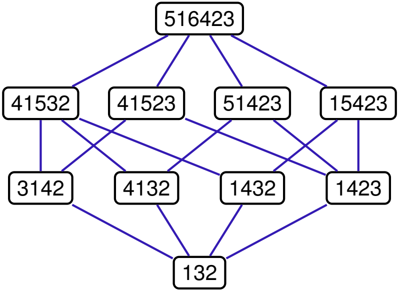
- A peak is an occurrence of 132 or 231, e.g., 13415.
- A permutation is alternating (up-down or down-up) iff it avoids 123 and 321 as consecutive patterns.
- Elizalde–Noy (2003), Aldred, Amigó, Atkinson, Bandt, Baxter, Bernini, Bóna, Dotsenko, Duane, Dwyer, Ehrenborg, Ferrari, Keller, Kennel, Khoroshkin, Kitaev, Liese, Liu, Mansour, McCaughan, Mendes, Nakamura, Perarnau, Perry, Pompe, Pudwell, Rawlings, Remmel, Sagan, Shapiro, Steingrímsson, Warlimont, Willenbring, Zeilberger, . . .

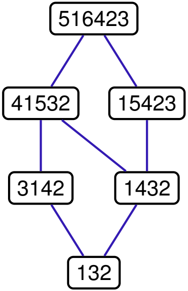
Pattern posets

Pattern order: order permutations by pattern containment.
$\sigma \leq \tau$ if σ occurs as a pattern it τ.

Classical pattern



Consecutive pattern



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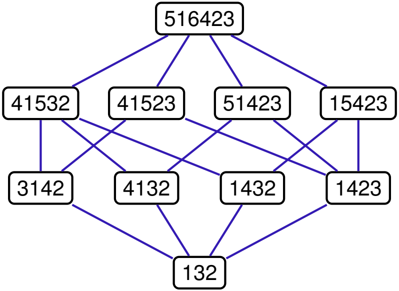

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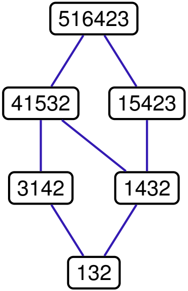
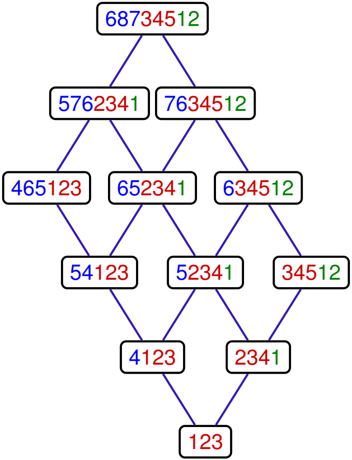


Consecutive pattern poset is more manageable.

- Consecutive case: every permutation covers at most two others.
- Wilf (2002): Möbius function $\mu(\sigma, \tau)$ of the pattern poset?
  - Known only in consecutive case: Bernini–Ferrari–Steingrímsson, Sagan–Willenbring (2011).

Consecutive pattern poset

When σ occurs just once in τ,
$[\sigma, \tau]$ is a product of two chains [BFS11].



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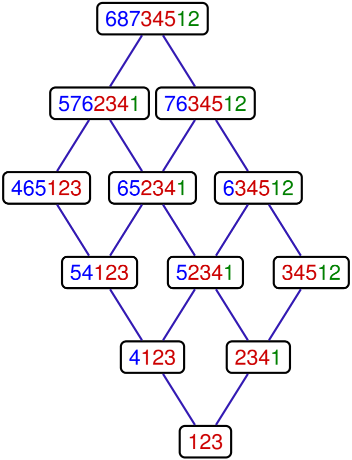
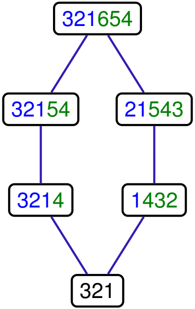


Classical case: wide open even in this special case.

Main questions

Unless otherwise specified: consecutive pattern poset.



1. Which open intervals are disconnected?
2. Which intervals are shellable?
3. Which intervals are rank-unimodal?
4. Which intervals are strongly Sperner?
5. Which intervals have Möbius function equal to 0?

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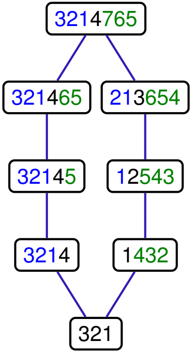
Definition. For $\sigma < \tau$, we say that σ straddles τ if σ is both a prefix and suffix of τ and has no other occurrences in τ.



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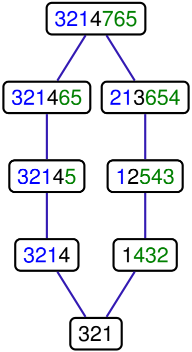

Theorem [Elizalde, McN.]. For $\sigma < \tau$ with $|\tau| - |\sigma| \geq 3$, we have that the open interval (σ, τ) is disconnected if and only if σ straddles τ. In this case, (σ, τ) consists of two disjoint chains.

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Some combinatorial topology...

$$\text{Poset } P \longrightarrow \text{Simplicial complex } \Delta(P)$$

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Order complex of $[p, q]$: faces of $\Delta(p, q)$ are the chains in (p, q).

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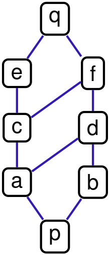

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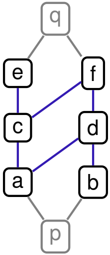

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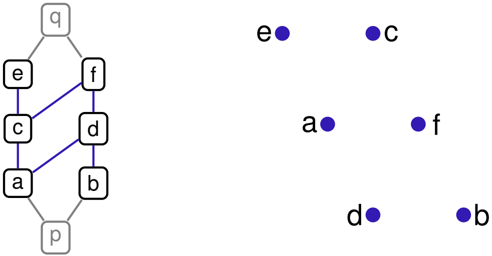

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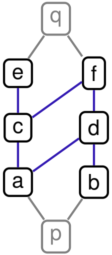

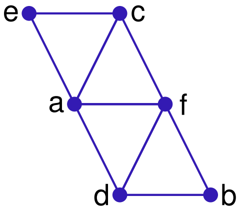

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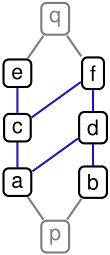

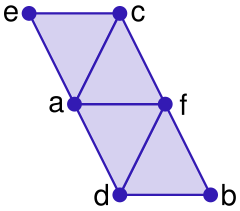

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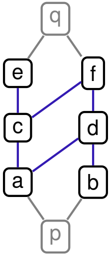

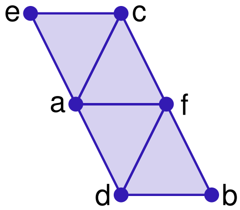

Definition. A pure d-dimensional complex is shellable if its facets can be ordered $F_1, F_2, \ldots, F_n$ such that, for all $2 \leq i \leq n$,

$$F_i \cap (F_1 \cup F_2 \cup \cdots \cup F_{i-1})$$

is pure and $(d-1)$-dimensional.

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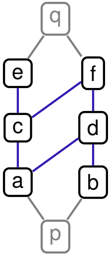

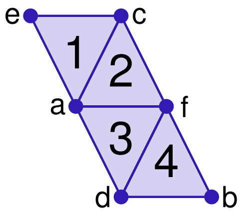

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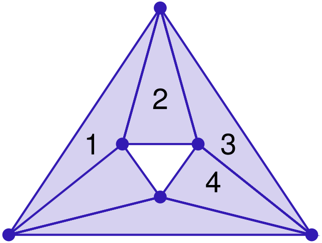

Shellability

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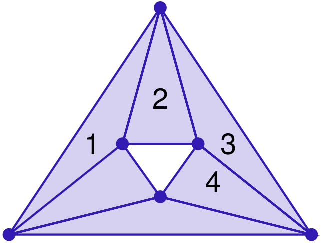
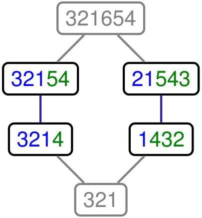


Why we care about shellability:

- If $\Delta(p, q)$ is shellable, then it's either contractible, or homotopic to a wedge of $|\mu(p, q)|$ spheres in the top dimension.
- Combinatorial tools for showing shellability of $\Delta(P)$: EL-shellability, CL-shellability, etc.

Disconnected and non-shellable

Main non-shellable example. (p, q) disconnected with $d \geq 1$: $\Delta(p, q)$ is not shellable.



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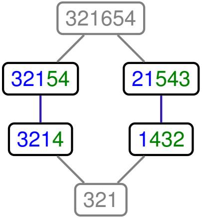

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Skew sum: $21 \ominus 3214 = 653214$.

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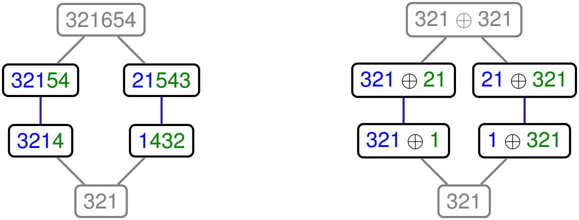

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Lemma. If π is indecomposable with $|\pi| \geq 3$, then $\Delta(\pi, \pi \oplus \pi)$ is non-trivially disconnected and so not shellable.

Almost all intervals are not shellable

Theorem [McN. & Steingrímsson; Elizalde & McN.].
Fix σ. Randomly choosing τ of length n,

$$\lim_{n\to\infty} (\text{Probability that } \Delta(\sigma, \tau) \text{ is shellable}) = 0.$$

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Idea of proof.

- Björner: If $[\sigma, \tau]$ is shellable (i.e. $\Delta(\sigma, \tau)$ is), then so is every subinterval of $[\sigma, \tau]$.
- Thus, if $[\sigma, \tau]$ contains a non-trivial disconnected subinterval, then it can't be shellable.
- Show every $[\sigma, \tau]$ as $n \to \infty$ contains $[\pi, \pi \oplus \pi]$ with π indecomposable,
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What's the good news?

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We know: if $[\sigma, \tau]$ contains a non-trivial disconnected subinterval, then $[\sigma, \tau]$ is not shellable.

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Example. $[21,\ 12\cdots r \oplus 21 \oplus 21 \oplus \cdots \oplus 21 \oplus 12\cdots s]$ is shellable.

Idea of proof. Show $[\sigma, \tau]$ is dual CL-shellable.

A related shellability result

Theorem [Elizalde & McN.] The interval $[\sigma, \tau]$ is shellable if it contains no open subinterval consisting of two disjoint chains of length ≥ 2.

Theorem [Billera & Myers, '99] Any poset is shellable if it contains no induced subposet of the form 2+2.

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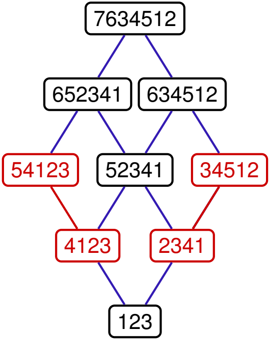
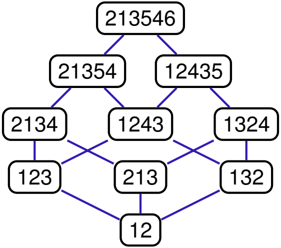
Has 2+2 as induced subposet, but
has no open subinterval consisting of two disjoint chains
of length ≥ 2:



3. Which intervals are rank-unimodal?



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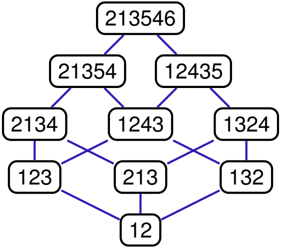

Rank sizes: 1, 3, 3, 2, 1.

So $[\sigma, \tau]$ is not rank-symmetric.

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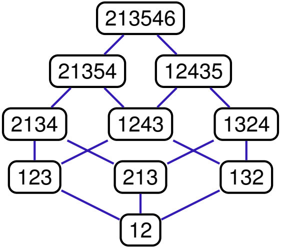

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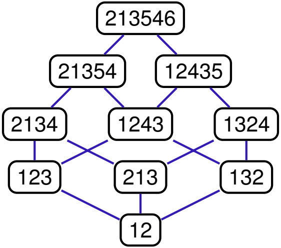

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- Top part is grid-like.
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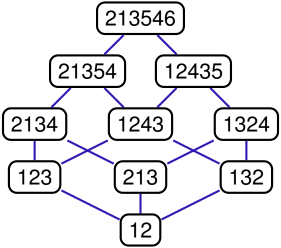

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- Top part is grid-like.
- Use explicit injection for all other ranks.

Conjecture [McN. & Steingrímsson] Every interval $[\sigma, \tau]$ in the classical pattern poset is rank-unimodal.

True for intervals of rank ≤ 8.

4. Which intervals are strongly Sperner?

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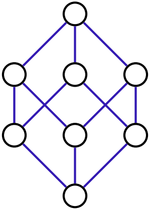
Definition. A poset P is Sperner if the largest rank size equals the size of the largest antichain.
In other words, some rank level is an antichain of maximum size.

Example.

Sperner:



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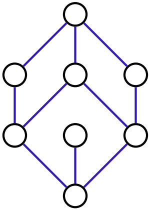
Not Sperner: 

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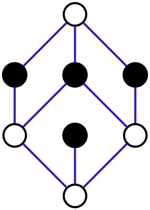
Not Sperner: 

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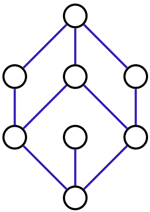
Not Sperner: 

A k-family is a union of k antichains.

Definition. A poset P is k-Sperner if the sum of the sizes of the k largest ranks equals the size of the largest k-family.
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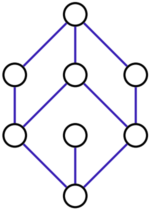
Example.

Not Sperner:
but 2-Sperner:



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Definition. A poset P is k-Sperner if the sum of the sizes of the k largest ranks equals the size of the largest k-family.
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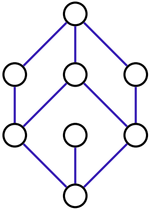
Example.

Not Sperner:
but 2-Sperner:



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In other words, some set of k rank levels is an k-family of maximum size.
P is strongly Sperner if is it k-Sperner for all k.

4. Which intervals are strongly Sperner?

Theorem [Elizalde & McN.] Every interval $[\sigma, \tau]$ is strongly Sperner.

Idea of proof.

- A 1980 result of Griggs gives a condition equivalent to strongly Sperner for rank-unimodal posets.
- We prove this condition, using the injections from our rank-unimodality proof.

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Interior $i(\tau)$: the permutation pattern obtained by deleting first and last element of τ.
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Examples.
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$\tau = 123456$ (monotone), $x(\tau) = 12345$
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Theorem [BFS, SW (2011)]. For $\sigma \leq \tau$,

$$\mu(\sigma,\tau) = \begin{cases} \mu(\sigma, x(\tau)) & \text{if } |\tau| - |\sigma| > 2 \text{ and } \sigma \leq x(\tau) \not\leq i(\tau), \\ 1 & \text{if } |\tau| - |\sigma| = 2, \tau \text{ is not monotone,} \\ & \text{and } \sigma \in \{i(\tau), x(\tau)\}, \\ (-1)^{|\tau|-|\sigma|} & \text{if } |\tau| - |\sigma| < 2, \\ 0 & \text{otherwise.} \end{cases}$$

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Note. $x(\tau)$ plays a crucial role.

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Answer. Almost all of them.

Theorem [Elizalde & McN.] Fix σ. Randomly choosing τ of length n with $\tau \geq \sigma$,

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Length of the exterior

$n\backslash k$	1	2	3	4	5	6	7	8	9
2	2								
3	4	2							
4	12	10	2						
5	48	58	12	2					
6	280	306	118	14	2				
7	1864	2186	822	150	16	2			
8	14840	17034	6580	1660	186	18	2		
9	132276	154162	58854	15118	2222	226	20	2	
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Open probems

Open probems: exterior

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1. Find a formula for the entries in the table.
2. Known: $\#\{t \in S_n : |x(\tau)| = 1\} \equiv 0 \bmod 4$.
   True? $\#\{t \in S_n : |x(\tau)| = 2\} \equiv 2 \bmod 4$.
3. For each k, find $\lim_{n\to\infty} \mathbb{P}_n(|x(\tau)| = k)$. (Know limit exists.)
   Bóna: $0.3640981 \le \lim_{n\to\infty} \mathbb{P}_n(|x(\tau)| = 1) \le 0.3640993$.
4. Find the exact value of $\lim_{n\to\infty} \mathbb{E}_n(|x(\tau)|)$.

Open problems: pattern posets

Consecutive case:

5. Characterize those intervals $[\sigma, \tau]$ that are lattices (in terms of easy conditions on σ and τ).
6. Find an easy classification of intervals that contain no non-trivial disconnected subinterval (and are thus shellable).

Classical case:

7. The question that started it all: what's the Möbius function $\mu(\sigma, \tau)$?
8. Prove the rank-unimodality conjecture.
9. Can anything be said about when σ occurs just once in τ?
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11. Find a good way to test shellability by computer.

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Thanks!